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SCHOOL SCIENCE
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IN ONE INCH ON THE MAP

THELMA WADDLE
H. C. Frick School, Pittsburgh, Pa.

The map of the world has held sway too long as a general, specific and all-purpose tool. Students are frequently urged to give detailed geographic information from this type of map on which only generalized material can be presented. Such misuses of a world map are due in large part to the fact that the limitations of scale are not realized. These limitations arise from the necessity of showing long distances and vast areas in a small space. Fundamental in the wise use of the globe and world map is a thorough appreciation of the scale employed.

We may more readily appreciate this matter of scale by noting specific examples. On the 16-inch globe one inch represents about 500 miles. Most wall maps of the world are constructed on approximately the same scale. The distance from New York City to Toledo, Ohio, is about 500 miles. When we realize the great variety of topographic features, the innumerable elevations and countless scenes we would see in an auto trip between these two cities we realize that only a continuous moving picture could tell the story. On the globe or world map the entire distance is shown by only one inch. Therefore, only the major geographic features can be included in this one-inch generalization.

The problem of areas is even more striking than that of distances. Let us suppose that we could have a world map on which every square inch equalled one square mile on the earth's surface. Chicago covers approximately two hundred square miles. Then on this hypothetical world map we would be trying to show the

entire city in two hundred square inches. Think of walking over just one square mile of the city and noting carefully the many things we would see. It would require numerous views to give even a representative picture of this two hundredth part of Chicago. Then on one square inch on our hypothetical map we would be attempting to picture this one mile section. Immediately we can see the limitations of a map on a scale even as large as this. However, we have not looked beyond our own doorstep. What would be the size of a sheet required to show the entire world on this scale? We would need a map containing more than 7000 square feet and obviously one much larger than school room walls. In comparing this huge hypothetical map with the world maps in general use we can readily understand that on the latter Chicago's allotted space would be negligible.

On the 16-inch globe or on a world map on a similar scale one square inch equals 250,000 square miles. What happens to the State of Illinois on this world map? The area of the state is approximately 57,000 square miles, therefore, on the map it would be shown in less than .2 of one square inch!

Perhaps it would be well to notice just how much of the United States could be shown in one square inch on this map. The combined areas of the six New England states plus those of New Jersey, Delaware, New York, Pennsylvania, Ohio and Indiana is approximately 250,000 square miles. When we realize that on a world map we are showing an area the size of all these states in one square inch we can understand why the globe and world map must be so greatly generalized.

What does this generalization do to a large European country such as France? One square inch would represent an area 37,000 square miles larger than the entire area of that country. Asia, the largest continent, with an area of seventeen and one-half million square miles would be represented in about seventy square inches. By such examples we realize that anything other than generalization is impossible on the world map. How much detail could possibly be shown about 250,000 square miles in one square inch?

NEW COMPOUNDS

Organic chemists of the world produced 25,000 new chemical compounds last year according to a survey made for the American Chemical Society. Compounds involving the element carbon predominated.

MATHEMATICS IN THE TELEPHONE INDUSTRY*

EDWARD C. MOLINA
Switching Theory Engineer, Bell Telephone Laboratories

Way back in 1894 Miss Pompelli, my algebra teacher in the Flushing High School, patiently expounded and reëxpounded the rules for obtaining the *coefficients* in the product of two or more polynomials.

You known how the rules run: Pick, in every possible manner, a term from each of the factors and multiply together the terms so picked to form a term of the product. Then collect together all like terms. For example

$$(x+x^2+x^3+x^4+x^5+x^6)(x+x^2+x^3+x^4+x^5+x^6)(x+x^2+x^3+x^4+x^5+x^6)$$
$$= (xxx)+(xxx^2+xx^2x+x^2xx)$$
$$+(x^3xx+xx^3x+xxx^3+x^2x^2x+x^2xx^2+xx^2x^2)$$
$$+(\text{etc.})$$
$$= x^3+3x^4+6x^5+\cdots.$$

Was Miss Pompelli's patience with the not too attentive class sustained by a vision of the role to be played by these coefficients 1, 3, 6, . . . in what has since become a giant industry? If so, perhaps the Victorian restrictions, in the Gay Nineties, on the intellectual activities of woman obliged her to withhold the vision from us. Let me recreate it for you, to be handed on to the present generation of pupils.

Consider these three dice. If we throw one of them up in the air it will land on the table with some one of its six sides facing toward the ceiling. The number on this side will be 1 or 2 or 3 or 4 or 5 or 6, any one of the figures being equally likely.

Now suppose we throw the three dice up in the air simultaneously; what is the probability that when they land on the table the *sum* of the three numbers facing up has an assigned value? This value, of course, may be anything from a minimum of $3=1+1+1$ to a maximum of $18=6+6+6$. If we take the trouble to set up the dice in all possible ways we will find just *one* combination giving the sum 3, *three* combinations result in the sum 4, *six* combinations in the sum 5, etc. But these numbers of

* A talk to the Chicago Teachers' Mathematics Club, February 16, 1940.

combinations, namely 1, 3, 6, . . . , are identical with the *coefficients* 1, 3, 6, . . . of the third, fourth, fifth, . . . powers of x in the product of Miss Pompelli's three polynomials.

To make a long story short, the *number of ways* we can construct an assigned sum, say S, by setting up the three dice in all possible ways is exactly equal to the *coefficient* of x to the power of S in the product of the three polynomials; this coefficient when divided by 216 (the total possible number of combinations with three dice) gives the probability that the sum will be S.

If you have grasped the plot of this short story you are well on your way to becoming telephone engineers because the dice problem is the analogue of an entire family or genus of telephone problems.

The mathematics we actually use in the solution of this genus of problems is usually as far removed from simple algebra as Shakespeare's verse differs from a nursery rhyme. But the expanded polynomial product contains the *basic philosophy* of the matter, and constitutes the first rung in the ladder leading up to Laplacian "generating" functions, Fourier analysis, the Dirichlet Multiple Integral theorem, etc.

I said that the dice problem is the analogue of a whole genus of telephone problems. A particular species of the genus arises from the recurrent or periodic structure of certain kinds of equipment. In long telephone circuits, for example, such a structure results from "transpositions" in the pair of wires of a talking circuit in order to eliminate crosstalk between adjacent pairs. Another example is that of a circuit wherein "repeaters" for amplifying the voice currents are inserted at several points along the line.

Now in any recurrent structure one is confronted with irregularities; no two transposed sections are exactly alike in length or in the distance between adjacent pairs; no two repeaters are exactly alike in their electrical characteristics or there may be slight variations between the repeaters in the voltages of the local batteries which supply them with current. The central question in this class of problems is: What is the probability that the cumulative effect or *sum*, of several independent irregularities will lie within a specified range? It is to be understood that each individual irregularity is likely to have any one of a finite, or infinite, number of possible values.

This class of problems is of primary importance to the transmission engineer. It is also of importance to the manufacturer of

telephone equipment since the transmission requirements set the tolerance limits for each and every piece of apparatus involved in a long distance connection.

Trunking Theory

Let us go back to the old algebra class and see what is going on.

That $(q+xp)^3=q^3+3q^2px+3qp^2x^2+p^3x^3$

has just been proven and the teacher is leading up to the general theorem

$$(q+xp)^n=\sum_{s=0}^{n}\left[\binom{n}{s}q^{n-s}p^s\right]x^s.$$

The coefficient $\binom{n}{s}q^{n-s}p^s$ of x^s in the expanded binomial is the compass which steers the trunking engineer through the labyrinth of switches and interconnecting links which constitute the modern dial telephone system.

You know that, in the special case of $q=1-p$, the coefficient gives the probability of an event happening exactly s times in n trials, the probability of the event happening in one trial being p. To the trunking engineer it gives the probability that, at a particular instant of time s out of a group of n subscribers are talking simultaneously.

Imagine a telephone central office serving 2000 subscribers; call this office A. To simplify the discussion assume that all outgoing calls from A are made to subscribers in a nearby exchange which may be referred to as exchange B.

Shall 2000 interoffice trunk lines be installed between A and B so that the entire 2000 subscribers of A may talk simultaneously with their friends in B? Of course not; in the first place not once in a century would all the subscribers of A wish to talk simultaneously unless an earthquake, or some such catastrophe, were to occur; in the second place the cost of the installation would be intolerable.

A more rational and quite practical point of departure for determining the number of trunks required for giving adequate service at a reasonable price is: Install such a number of trunks that, under normal conditions, only about one call per hundred shall fail to obtain immediately an idle trunk.

With this point of view in mind suppose that during the busy hour of the day each of the 2000 subscribers of exchange A

makes a call for B, the calls being placed at random with reference to time during the hour under consideration. Suppose, moreover that, when a connection is established between the two exchanges, the conversation lasts for exactly two minutes. Since 2 minutes is 1/30 of an hour, it is anticipated that *on the average* 1/30 of 2000 = 66.7, say 67, simultaneous conversations must be provided for. But what about deviations from the average; more particularly, what are the probabilities that the average number will be exceeded to various extents? A glance at our probability tables indicates that to meet the desired "1 in 100 grade of service" we should install between the two offices $87 = 67 + 20$ trunk lines.

Let us carry this trunking problem a step further.

Manual or automatic means are required in order that any one of the 2000 subscribers shall have access to all of the 87 outgoing trunks. The mechanics of this part of the problem would be simpler and perhaps cheaper if the subscribers were divided into 2 groups of 1000 each and then 2 separate groups of trunks running from exchange A to exchange B were installed. To what extent will this arrangement, if put into effect, increase or decrease the total number of trunks? Consider the following analysis.

On the assumptions made above each group of 1000 subscribers will contribute on the average $66.7/2 = 33.4$ simultaneous conversations. When the average is 33.4, the probability table tells us that 48 trunks are required to give a 1 in 100 grade of service. Therefore, it will be necessary to install 48 trunks per group of 1000 subscribers or a total of $2 \times 48 = 96$ trunks. This is an excess of $96 - 87 = 9$ trunks over the number required when it is feasible to serve the subscribers as a single group of 2000.

You will appreciate without further discussion that mathematics, more particularly the mathematical theory of probability, together with a knowledge of unit cost figures and other factors which the telephone engineer must take under consideration, enable him to determine the optimum dialing system to be installed.

The simple trunking problem we have just discussed is solved by use of the binomial summation. When we pass to complex problems more powerful mathematical tools come into play. It will suffice to mention the Erlang Statistical Equilibrium attack depending on finite difference equations and Crommelin's combined use of Laplacian generating functions and the theory of complex variables.

Returning for a moment to the binomial I would like to point out the striking illustration it affords of the desirability of a knowledge of higher mathematics.

It happens, as Laplace has shown, that the incomplete binomial summation is equal to the ratio of two beta integrals. In other words we have the exact equation

$$\sum_{s=c}^{n}\binom{n}{s}p^s(1-p)^{n-s}=\int_0^p x^{c-1}(1-x)^{n-c}dx\Big/\int_0^1 x^{c-1}(1-x)^{n-c}dx.$$

Now, believe it or not, for many ranges of values of p (n and c being given) it is *very* much easier to compute the numerical values of the integrals than the values of the simple algebraic terms. Moral: The student of the calculus should avoid the tradition to the effect that the mathematician's sole interest in integrals is to express them in simple algebraic forms.

Also, before presenting for your consideration another species of telephone problems, permit me to state that the mathematician who specializes on trunking problems does not thereby isolate himself from contact with other fields of applied mathematics. This contact arises in many ways as is illustrated by the following facts.

The trunking theory mathematician makes constant use of the Poisson Exponential Binomial Limit $a^s e^{-a}/s!$, where $a=np$ is the average number of times an event of probability p will happen in n trials. Now this Poisson Limit makes its appearance in several domains of mathematical physics and related sciences. For example, Bateman, in 1910, found that the distribution around the average number of alpha particles striking the screen in the classic experiment of Rutherford and Geiger is of the Poisson form. Moreover, recently my young collaborator, Mr. R. I. Wilkinson, was invited to present a paper on the characteristics of the Poisson frequency distribution with special reference to its bearing on the physico-biological problem of killing bacteria by radiations.

Sampling

So far we have only dealt with algebra although the higher mathematics have been mentioned. Let us now discuss a problem involving explicitly the integral calculus. The problem is chosen from the extensive class designated sampling.

We will first discuss the problem in terms familiar to all of us. It will then suffice to merely restate it in the equivalent telephone terms.

Our main building at 463 West Street, New York, houses a population of approximately 4000 individuals. In order to serve this population the building is equipped with several elevators. As a matter of safety there is a limit, of course, to the number of persons allowed to ride simultaneously in any one of the elevators. Suppose this limit is 20. The question arises as to the aggregate weight of a random group of 20 out of 4000 individuals. Obviously it is impractical to weigh each one of them before entering the elevator to make sure that the carrying weight per square foot of the elevator floor is not exceeded.

We can answer this question in a probability sense if we know the law of distribution of weights for the entire population of the building. Let $f(x)dx$ be the number of persons whose weights lie in the small range from $(x-dx/2)$ to $(x+dx/2)$ and assume that the form of the function is of the Pearsonian type III, namely $f(x)=Kx^s e^{-kx}$. By referring to any standard textbook we will find for the probability that the aggregate weight of 20 passengers (selected at random from a large population) lies in the range $(w-dw/2)$ to $(w+dw/2)$, the multiple integral of the twentieth order

$$\int\int\int \cdots \int f(x_1)f(x_2)f(x_3) \cdots f(x_{20})dx_1dx_2dx_3 \cdots dx_{20}$$

the limits of integration being such that

$$(w-dw/2)<x_1+x_2+x_3+ \cdots x_{20}<(w+dw/2).$$

Bearing in mind that $f(x)=Kx^s e^{-kx}$, all that remains to be done is to turn the pages of Todhunter's *Integral Calculus*, or of any other which you may prefer, until we arrive at the section on the Dirichlet multiple integral. We obtain for the desired probability that the combined weight of the sample of 20 individuals is in the neighborhood of w,

$$F(w)dw=(K_1x^{s_1}e^{-k_1x})dw$$

where, writing n for 20 (since the relations are not limited to 20)

$$s_1=n(s+1)-1$$

$$k_1=k$$

$$K_1=[K\Gamma(s+1)]^n/\Gamma(ns+s).$$

So much for the mathematics of our elevator analogue of the telephone problem. Now for the statement of the latter.

A telephone office serves 10,000 subscribers. The service demand curve for the 10,000 individuals collectively is known. By this I mean that we know the average value of the amount of time a subscriber wishes to talk and that we also know what proportion of the 10,000 will each talk for any assigned time above or below the average.

Now suppose that it is proposed to install a new dialing system in the office under consideration. Suppose, moreover, that the new system requires the division of the 10,000 subscribers' lines into small groups of, say, 20 lines per group.

Question: What is the probability that a group of 20 lines, chosen at random from the totality of 10,000 will demand service exceeding by any specified per cent 20 times the average demand per line?

This question is obviously equivalent to the one which was answered in connection with the elevator service problem.

In sampling problems one must distinguish between those which are *a priori* (direct) and those which are *a posteriori* (inverse). The first class embraces problems wherein the characteristics of a population, or universe *are given* and one asks for the probabilities that a sample *not yet* taken will possess such and such properties. In the second class one is presented with the results derived from a sample which *has been* taken from a population whose parameters are *unknown*. In this case the probabilities in favor of various sets of values for the unknown parameters are required.

This differentiation relates to the probability principles involved in the problem to be solved. Another differentiation is called for which relates rather to the mathematical technique which one must apply. The sampling of *attributes* is under consideration when each individual should answer "yes" or "no." For example, when the question is "what per cent of the population of Chicago is native," each individual responds "I am" or "I am not." But one deals with the sampling of *variables* if, for example, the age distribution is desired. In this case an individual is not merely young or old; he, or she, is 23.5 years old or 37.6 years old, etc.

These four types of sampling problems, namely, "direct-attributes," "direct-variables," "inverse-attributes" and "inverse-variables" are encountered in all branches of the telephone industry. The particular problem we considered is of the "direct-variables" type.

Conclusion

The thesis assigned for the talk this evening was the extent to which mathematics enters in the telephone industry.

That we use a great deal of mathematics has, I trust, been made clear by the problems submitted for your consideration. Naturally, illustrations have been chosen only from those fields of telephone engineering work with which I am in direct contact. To obtain a complete picture, in its proper perspective, recourse must be had to the *Bell System Technical Journal*. There you will find how transmission engineers, for example, use advanced mathematics such as the Heaviside operational calculus, integral equations, function theory, etc.

You will be particularly interested in the paper by Dr. G. A. Campbell entitled "Mathematics in Industrial Research" published in the Oct., 1924 issue of our *Journal*. I would also call attention to the paper by Dr. H. W. Bode of the Bell Laboratories which appeared in the April, 1938 number of the *Journal of Engineering Education*. One who reads this paper entitled "Mathematical Fundamentals for Modern Electrical Engineering" may be assured that the author would have been entirely justified in substituting "telephone" for "electrical" in the caption under which he wrote.

I will close this talk in the University wherein Prof. Emeritus L. E. Dickson wrote his monumental work on the *Theory of Numbers* by calling your attention to a paper by Mr. H. P. Lawther, Jr., of the Southwestern Bell Telephone Co. You will find it in the April, 1935 issue of the *Bell System Technical Journal* under the title, "An Application of *Number Theory* to the Splicing of Telephone Cables."

NOTE ON A DERIVATION OF THE QUADRATIC FORMULA

Adrian Struyk
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In connection with the article by B. L. Waits in the February, 1940 issue of School Science and Mathematics it should be noted that the derivation there given is not general, but is valid only for a, b, c real, x complex.

For x real, $x = x_1 + iy$, makes $y_1 = 0$. This in equation (2) of the article mentioned, prevents the inference $x_1 = -b/2a$. Equation (1), thus reduces to the original form in which the roots are real.

A READER FOR MICROFILM

HERMAN BRANSON
Dillard University, New Orleans, La.

Many teachers in high schools and colleges have felt keenly the need of adequate references in working on papers or in their class work. The coming of microfilm copies has promised relief, but there are still many problems to be solved before this technique can realize more than a fraction of its potential use. The initial problem of production and distribution of microfilm has been met partially by governmental agencies, libraries, and commercial establishments; and recently the American Mathematical Society has announced an excellent microfilm service at nominal rates to subscribers to *Mathematical Review*.[1] Hence, the chief problem confronting the teacher is how to read his microfilm.

The 35 mm projectors can be used. Unless they have been remodeled, however, they heat up badly, causing the film to buckle. In addition, they are inconvenient to use since they require at least a partially darkened room for good results. And, above all, the projectors are relatively expensive. Projection reading devices sold commercially are also too expensive for the average teacher and even the small school. The teacher has either to devise his own apparatus or to order some inexpensive commercial magnifier.

The author met this leading problem by constructing a visual magnifier from a commercial binocular viewer for 35 mm film strips at a total cost of approximately 80 cents.

The apparatus is built around the viewer which can be bought at a novelty store for about 50 cents. The magnification of approximately 4× is sufficient for 35 mm single or double frame microcopy from the average book and periodical. The viewer has a film propelling lever and the carry track holds the film only on the edges, thus reducing the danger of scratching. The film passes just behind a translucent window. One window is mounted over a hole cut in the bottom of a cigar box. A 15 watt bulb is mounted behind the hole and a white cardboard reflector is placed so as to illuminate the translucent window

[1] Yet even with these services the individual worker often has need of photographing equipment for microfilm copies. The author hopes to describe the apparatus devised for producing 35 mm single frame microfilm copies either in black and white or in color in another note to this journal.

evenly. A hole is cut in the side of the box for the electric cord and the lid is tacked down.

The photograph shows the completed reader in use. It is portable and can be used without inconvenience to others. The light has been left on for over 4 hours without any over heating and film buckling.

The microfilm[2] shown in the picture are negatives prepared here. The staff feels that the reading of the negative (clear

letters on black background) is less fatiguing than the reading of the positive (black letters on clear background). The author has experienced little fatigue in using the reader continuously for periods up to an hour. With longer use broken by note-taking and calculating, the author has felt practically no fatigue.

The enthusiastic praise by members of the staff of a roll of single frame 35 mm color pictures in the reader suggests its value for biologists, ornithologists, and others in whose work inexpensive color copies are of value.

[2] The microfilm is stored in small metal containers with identifying labels. The cans are stored in a cigar box with cardboard spacers. A bound notebook carries complete data. The author finds it convenient to photograph the title page as the first picture on each roll, regardless of the number of articles taken from the periodical or book.

THE ORIENTAL COCKROACH AS A SOURCE OF LIVING PARASITES FOR GENERAL ZOOLOGY CLASSES

PAUL. A. MEGLITSCH
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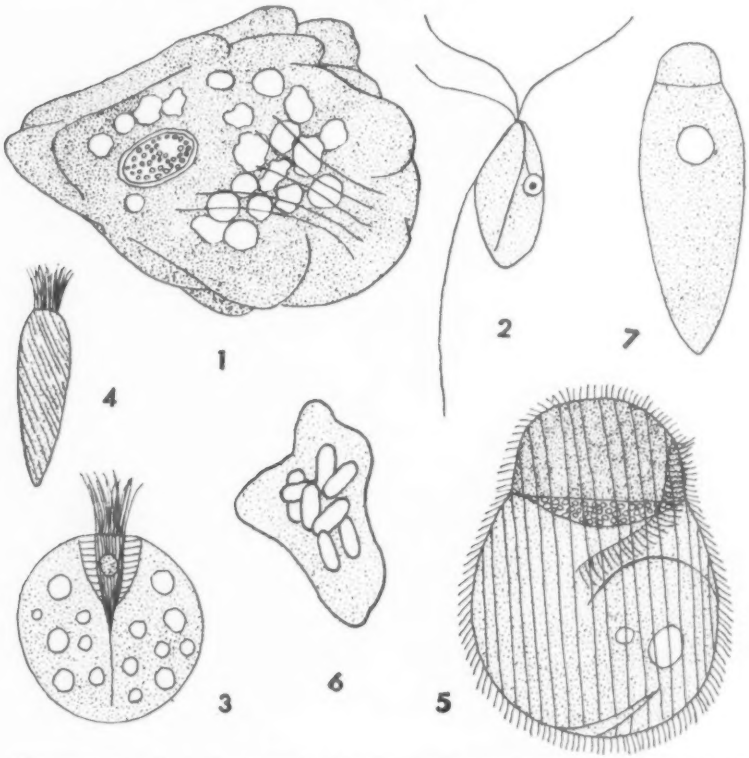
Very few general zoology classes have an opportunity to study living parasites or commensals. As a result of this the students have difficulty in visualizing the parasitic habitat and mode of life. Several important reasons for omitting a study of living parasites may be enumerated. Without expensive equipment for keeping the host animals alive in the laboratory, it is difficult to obtain an adequate supply of parasites from living animals. Usually the low incidence of infection is a further difficulty, since it leads to much loss of time spent in opening sterile hosts or hosts with few parasites. In addition, the dissection of more highly specialized living forms in beginning courses leads to aesthetic disturbances on the part of some students.

The common oriental cockroach, *Blatta orientalis*, is an excellent host species which offsets the major share of the difficulties mentioned above. It is small and can be kept in the laboratory with a minimum of equipment and care. After several weeks of laboratory culture the incidence of infection and number of individual parasites occurring in each host are very high. Very little time is required for dissection. Dissection can be completed before the class begins, or in class, with little aesthetic or emotional difficulties. Because of these qualities, the oriental cockroach can be used for general zoology classes with a minimum of lost time and with a restricted amount of equipment.

Cockroaches are common and widely distributed throughout the country. In some localities, the American cockroach is more common than the oriental. It can be substituted for the oriental cockroach, although the intestinal fauna is usually somewhat more restricted. The small German cockroach, while useful if no other species can be obtained, is not recommended. Roaches may be caught in basements or around buildings at night when they come out to feed. Simple traps can be devised to capture the insects. A single battery jar will suffice to keep fifty or more insects. Somewhat crowded conditions are desirable as this leads to rapid cross-infections through contamination of food by infec-

tive cysts. The best food is crushed Fleishman's yeast cake. Potatoes and apples cut into small cubes may also be used. Water is most easily administered in vials plugged with cotton and laid on the bottom of the container.

After several weeks, the insects are ready for use. The dilated posterior end of the alimentary tract, the colon, is the best source of parasitic (commensal) protozoans. Dissection of anaes-



TEXTFIGURE I. 1. *Endamoeba blattae.* 2. *Retortomonas blattae.* 3. *Lophomonas blattarum.* 4. *Lophomonas striata.* 5. *Nyctotherus ovalis.* 6. *Coelosporidium periplanetae.* 7. *Gregarina blattarum.* (Adapted from Kudo)

thetized specimens is simple. The anterior end of the body is held with one pair of forceps and the last segment of the abdomen is grasped with a second pair. The colon will cling to the last segment as it is pulled away from the body. The colon will usually break off at the anterior end, where it joins the mid-gut. If the pressure is applied slowly and regularly the long, yellowish Malpighian tubules surrounding the mid-gut will usually be removed with the colon.

The colon, with the Malpighian tubules adhering to it, is placed in a watch glass containing Locke solution, diluted so that the NaCl content is 0.45% (100 cc. H_2O; 0.45 gm. NaCl; 0.02 gm. $CaCl_2$; 0.001 gm. $NaHCO_3$). The protozoa will remain alive for several hours in this solution, and may be given to the students if dissection of the host animals is carried out by the instructor before class. A half dozen well infected cockroaches will supply a class of twenty-five students with an abundance of material.

From the colon the following forms are commonly obtained.

Class Sarcodina

Endamoeba blattae. This is a common, large amoeba with an unusually large nucleus and thick nuclear membrane. The nucleus is filled with refractive spheres of achromatic material. Many starch grains and yeast cells occur in food vacuoles. Small refractive cysts with numerous indistinct nuclei frequently occur. Size: 50–200 μ in diameter. (Fig. 1)

Endamoeba thomsoni. This is a fairly common amoeba of medium size, with granular endoplasm. It undergoes rapid locomotion. The nucleus is smaller than that of *E. blattae* with a thin nuclear membrane. Size: 15–30 μ in diameter.

Endolimax blattarum. This is a very small amoeba with clear, transparent cytoplasm. The small nucleus is usually invisible or indistinctly visible in life. A few small food vacuoles may be found. Size: 3–15 μ in diameter.

Class Mastigophora

Retortomonas blattae. This is a very small flagellated protozoan of the order Protomonadina, characterized by two flagella, one larger and directed anteriorly and one smaller, directed posteriorly. The body is spindle shaped, sometimes almost oval. Size: 6–9 μ. (Fig. 2)

Hexamitus periplanetae. This is a small flagellated protozoan of the order Polymastigina characterized by eight flagella, six of which are directed anteriorly and two of which are directed posteriorly. Small, oval cysts are not uncommon. Size: 10–15 μ in length.

Lophomonas blattarum. This is a medium sized organism belonging to the order Hypermastigina. The body is spherical. A brush of flagella occurs at the anterior end, terminating a pro-

trusible bundle of axostylar filaments. The nucleus lies in a basket formed by these filaments just below the tuft of flagella. Binary and multiple fission occur and dividing forms are rather common. Spherical binucleate cysts are common. Size: 25–30 μ in diameter. Forms undergoing multiple fission sometimes reach 50 μ. (Fig. 3)

Lophomonas striata. The body is elongated and spindle shaped. It is supported by long refractive structures which are imbedded in the surface of the body. These may be a hyperparasite, *Fusiformis lophomonadis.* It forms oval cysts with two nuclei. The flagellar tuft is similar to that in *Lophomanas blattarum.* Size: 20–35 u in length. (Fig. 4)

Class Ciliata

Balantidium praenucleatum. This is a medium sized organism belonging to the suborder Heterotricha. An oval or ellipsoidal body is covered with rather long, uniform cilia. A small cytopharynx near the anterior end contains adoral membranelles. An oblong macronucleus containing large spheres of chromatin lies at the anterior end. A single micronucleus is attached to the macronucleus. A single contractile vacuole occurs at the posterior end, close to the anal pore or cytoproct. Size: 50–130 μ long.

Nyctotherus ovalis. This is another member of the suborder Heterotricha. The body is oval and flattened. The macronucleus is large, and divides the body into a darker anterior part containing glycogenous material and a larger, clearer posterior part. The adoral zone of membranelles is long. The mouth is near the center of the body. A long cytopharynx contains a distinct undulating membrane. A contractile vacuole lies near the cytoproct at the posterior end. (Fig. 5)

Class Sporozoa

There are two members of the class Sporozoa which also occur frequently, although not in the colon. In the long, yellowish Malpighian tubules there are usually specimens of *Coelosporidium periplanetae.* This is a very small Haplosporidian which will appear as a transparent amoeboid organism containing eight refractive spores. (Fig. 6) These occur in almost every cockroach and can be obtained easily by teasing apart some of the Malpighian tubules on a slide.

Another sporozoan has a rather high incidence of infection.

This is a very large gregarine, *Gregarina blattarum*, which sometimes attains a length of over 1 mm. in syzygy. (Fig. 7) They are typical cephaline gregarines occurring in the slender mid-gut, obtained by teasing the mid-gut apart on a slide or in a watch glass. A large nucleus is found in the deuteromerite. There is no nucleus in the protomerite. Further information concerning these forms may be obtained from any textbook on Protozoology. Kudo (1939) "Protozoology" discusses all of these forms.

A small nematode is frequently found in the colon. Because of its small size and transparent cuticle it offers an excellent opportunity for studying the structure of nematodes. The important organs are easily located under low or high power.

In addition to the visualization of the parasitic habitat there are several general principles which may be brought out by the study of the cockroach protozoa. The Endamoebae may be used to develop the concept of generic and specific differences. In general zoology, it is not always easy to demonstrate two species belonging to the same genus. The two species of Lophomonas almost always are found if several cockroaches are opened and also show most clearly the generic similarities and specific differences.

A series of typical colorless flagellates occur. These may be used to emphasize the developmental trend toward an increase in the number of flagella. In the Protomastigina there are but a few flagella, while in the Hypermastigina large numbers are found. The large ciliate, *Nyctotherus ovalis*, is a good example of a heterotrichous ciliate. It may be used as a step between Paramecium and Euplotes in a series to show the type of specializations which may occur among ciliates. The permanent cytoproct is easily visible and the adoral zone is distinct.

The writer believes that the study of cockroach parasites proves to be a valuable addition to the study of mixed cultures, serving to acquaint the student with a new habitat and to broaden his experience with the Protozoa.

LEAD USED TO SHAPE BAND INSTRUMENTS

Have you ever wondered how the amazing convolutions of band instruments—where the music goes round and round—are made. At a band instrument factory here melted lead is poured into the instrument tubing. When it has cooled the tube is bent properly and hammered into proper shape and the wrinkles taken out. Finally the lead is melted away and the instrument buffed and polished, without injury.

EXPERIMENTS FOR CHEMISTRY CLUBS

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It is the opinion of the writer that a successful chemistry club must offer its members a chance to demonstrate or see demonstrated experiments which can be described by some of the following adjectives: interesting, amusing, startling, unusual, entertaining, spectacular, etc. It is the object of this article to describe a number of such experiments that the author has used in his chemistry club.

It is not implied here that this type of demonstration experiment shall be the only activity of a chemistry club or that such experiments shall constitute the major portion of the program; rather they should be judiciously interspersed among other experiments, reports, films, etc. of a practical and educational nature.

None of the experiments are original, but the author has made certain changes in the strength of solutions and in the manner of performing the experiments as experience has shown gave greater success.

THE IODINE CLOCK

1. Two hundred cc. of starch emulsion are placed in a one liter round bottomed flask. To this, 2 cc. of a .5 N solution of iodic acid is added.

2. In another container place 100 cc. of water and 2 cc. of a .5 N solution of sulphurous acid. Shake the mixture in 1 thoroughly and the mixture in 2 thoroughly. Then pour the mixture 2 into the flask with 1 and again shake thoroughly. Clamp the flask to a ring stand in plain view of the audience and allow the mixture to stand for about 30 seconds and it will suddenly turn dark blue faster than you can wink an eye.

To insure success, the solutions should be made up very carefully. The starch emulsion is made by heating to boiling temperature a mixture of one liter of water and one tablespoon of the socalled "soluble starch": the resulting cloudy suspension is filtered. The filtrate should be reasonably clear as this makes the experiment more startling.

Sulphurous acid—Dissolve 26 grams of sodium bisulphite in enough water to make one liter of solution. This solution will be equivalent to a .5 N solution of sulphurous acid.

Iodic acid—First, approximately .5 N sulphuric acid is prepared by mixing 14 cc. of 95% H_2SO_4 S.G. 1.84 with enough water to make one liter of solution. Then mix 100 cc. of .5 N H_2SO_4 with 10.7 grams of potassium iodate. The iodic acid is formed as follows:

$$2KIO_3+H_2SO_4=2HIO_3+K_2SO_4.$$

The chemical reactions that take place in the flask are as follows:

(1) $$HIO_3+3H_2SO_3=HI+3H_2SO_4$$

(2) $$HIO_3+5HI=3H_2O+3I_2$$

(3) $$I_2+H_2SO_3+H_2O=H_2SO_4+2HI.$$

An excess of iodic acid was purposely added, so that as fast as hydroiodic acid is formed it is reacted upon by the excess iodic acid. However reaction (3) is faster than either (1) or (2) so that as long as the sulphurous acid lasts no iodine is set free. When the sulphurous acid is used up, the iodine is set free and gives the characteristic dark blue color when it reacts with the starch.

Burning Water

Fill an ordinary water pitcher with water from the tap. Drink some of the water to convince your audience that it is really water. Pour the water from the pitcher into a pneumatic trough containing a tablespoon of ether and a piece of potassium wrapped in a piece of filter paper. When the water reacts with the potassium, sufficient heat is generated to ignite the ether which burns on top of the water. The water appears to burn.

Beautiful Stars

Mount a candle in the bottom of a syrup can. For silver stars, ignite the candle, darken the room, and throw a *pinch* of magnesium or aluminum powder in the can. For golden stars use powdered iron. For a magic flash use lycopodium powder.

Self Lighting Cigarettes

Make a strong solution of white phosphorus in carbon disulphide. CAUTION: Don't make very much and keep what you do make in a glass stoppered bottle. This mixture is very inflammable. Put the end of the cigarette in this solution and wave it in the air. As soon as the carbon disulphide evaporates,

the phosphorus, which has a low kindling point, will ignite and the cigarette will appear to light itself.

Mysterious Restoration

Make a mixture of six parts of water to one part of alcohol. Soak a piece of cloth in the mixture, ignite the alcohol mixture, and the cloth will appear to burn but the fabric will not be destroyed.

Whiskey to Water

Tincture of iodine is diluted with water until it has the appearance of whiskey. Secure a whiskey bottle and pour into it the fake whiskey. To accomplish the apparent transformation a crystal of sodium thiosulphate, "Hypo," is concealed in a hole bored on the inside of the cork stopper. When the bottle is shaken the sodium thiosulphate reacts with the tincture of iodine and a colorless solution is produced.

Zoological Transformation

Make a white cardboard model of a tiger. Paint in the stripes with antimony chloride solution and lead acetate solution. Place the model under a large glass jar and pass in hyhrogen sulphide. The stripes which were painted with antimony chloride will turn orange, and those painted with the lead acetate will turn black. The chemical reactions involved are as follows:

$$2SbCl_3+3H_2S=Sb_2S_3+6HCl$$

$$Pb(C_2H_3O_2)_2+H_2S=PbS+2HC_2H_3O_2.$$

The Blushing Maiden

Secure a black and white photograph of a young lady and mount on cardboard. Paint the cheeks on the picture with phenolphthalein solution. When the picture is sprayed with ammonium hydroxide, the maiden will blush. An atomizer is used for the spraying.

The Magic Wand

The magic wand is a cylindrical piece of wood about 15″ long. Holes are bored in each end to conceal the chemicals which produce the color changes. Crystals of sodium carbonate are concealed in one end while crystals of tartaric acid are concealed in the other.

Stir a solution of phenolphthalein with the end containing the

sodium carbonate and the solution will turn pink. Then stir with the other end and the solution will again become clear.

Muddy Waters

Clear solutions of ferric chloride and sodium carbonate are mixed and the resulting liquid looks like muddy water.

Fire Burning under Water

Make a mixture of 5 parts by weight of potassium nitrate, 1 part of powdered charcoal, 2 parts of sulphur, and 1 part of strontium nitrate. Each chemical is powdered *separately* and then the chemicals are thoroughly mixed.

Procure a cardboard tube about 2″ long, closed at one end and shellac it inside and out. After the shellac is dry place the mixture in the tube. To perform the trick, ignite the powder, (Caution: When the powder is ignited the mouth of the tube should be held away from the face as the above mixture is essentially gunpowder) quickly place the mouth of the tube under water. The mixture burns very vigorously under the water and with a visible flame.

Lighting a Fire with a Piece of Ice

Place a piece of sodium in a piece of filter paper soaked in gasoline. Allow water to drip from a piece of ice so that it falls on the paper near the sodium. Paper will take fire.

Making Ice Burn

Place a piece of calcium carbide on the ice and ignite the acetylene which is generated.

Explosive Bubbles

Fill a clay pipe with cotton keeping the latter in place by inserting a screen at the mouth of the bowl. Saturate the cotton with gasoline. Dip the pipe in strong soap suds containing a little glycerine. Sail the bubble into the air and light with a candle or match. Bubble will explode and disappear in a brilliant flash.

Spontaneous Combustion

Mix one part by weight of potassium perchlorate and one part of powdered sugar. Touch the mixture with a glass rod on which has been placed a drop of concentrated sulphuric acid.

Shower of Sparks

Lead tartrate is placed in a test tube and heated until all the vapors are driven off. The resulting substance contains extremely minute particles of lead which will catch fire when the contents of the tube are poured into the air. If the tube is sealed after the lead tartrate has been heated, the trick can be performed at any time after the seal is broken.

Lighting a Candle with a Glass Rod

Mix equal parts of sugar and potassium chloride. Place some of this mixture on the wick of a candle. Touch with a glass rod on which has been placed a drop of concentrated sulphuric acid.

Surprise Cigarette

Soak some tobacco in potassium nitrate solution. Dry the tobacco, and roll a cigarette. When this cigarette is ignited it will burn very rapidly.

Magic Blotting Paper

Soak a blotter in potassium ferrocyanide and dry. Write with a solution of ferric ammonium sulphate. When the damp invisible is blotted blue writing appears.

Soak a blotter in ferric ammonium sulphate and dry. Write with a solution of sodium salicylate. When the damp invisible writing is blotted red writing appears. Other combinations are possible.

Disappearing Writing

Write with cobalt chloride on a plain sheet of white paper. The writing will be practically invisible. Warm the sheet of paper carefully over a burner. Blue writing will appear. Roll the sheet of paper into a tube so that the writing is on the inside and exposed. Breathe through the tube several times. Open up the sheet of paper and the writing is again practically invisible.

Chemical Joker

Put powdered potassium iodide in the bottom of a cardboard cylindrical box. Put powdered lead nitrate on top of the potassium iodide. Ask some one in the audience what color the powder is. Put the cover on the box and shake, the original white color changes to yellow.

Trick Matches

Dip heads of matches in sodium silicate (water glass). The matches when struck only sputter and go out.

Dip the lower four fifths of the match stick in sodium silicate. When these matches are lighted they burn down to the portion treated and go out.

Restless Moth Balls

Acetic acid, sodium carbonate and moth balls are placed in a glass bottle. The moth balls sink to the bottom, become covered with carbon dioxide bubbles, rise to the surface, lose the bubbles of gas, and sink. This process goes on continuously for quite a while.

Chemical Smoke-Producing Apparatus

To make chemical smoke the fumes of ammonium hydroxide and concentrated hydrochloric acid are mixed, a white smoke is produced which consists of finely divided ammonium chloride. An apparatus for producing smoke this way is pictured in the magazine *Popular Science*, Sept., 1937, p. 70.

Magic Seeds

The seeds are made up as follows: Take 6 g. of copper sulphate, 1 g. of ferrous sulphate, 1 g. of cobalt chloride, 6 g. of manganous sulphate, 4 g. of calcium sulphate and mix with enough water to make a paste. Mold the paste into seeds about the size of a bean. Make a solution of 2 ounces of sodium silicate in one glass of water. Drop seeds in the solution. A sort of chemical vegetation will grow if this mixture is allowed to stand over night.

Chemical Artillery

Put sodium carbonate and acetic acid in a bottle and cork. As soon as enough carbon dioxide is generated the cork is blown out.

The Diving Egg

An egg is placed in a glass cylinder containing dilute hydrochloric acid. The egg will sink and rise just like the moth balls in the Restless Moth Balls Experiment.

Change of Season

Secure a picture depicting a winter scene. Paint all the snow

with cobalt chloride. Tell the audience you are going to warm up the scene. Warm the picture over a Bunsen Burner. All the snow will turn green. Breathe on the picture and the green will disappear. Winter will return.

Magical Cardboard

Prepare a cardboard by rubbing it with dry ferric chloride. When you write on the cardboard with potassium sulphocyanide the writing will be red. When potassium ferrocyanide is used the writing will appear blue. When you write on the cardboard with tannic or gallic acid the writing will appear black.

The Changing Fluid

Cobalt when cool appears pink, when heated appears blue.

Magic Paper

A coarse grained paper is prepared by rubbing it with a dry mixture of equal parts of tannic acid and ferric ammonium sulphate. When you write on this paper with water, the writing appears black.

Magic Writing

Prepare a sheet of coarse grained paper with a dry mixture of sodium salicylate and ferric ammonium sulphate. When you write on this paper with water the writing will appear red.

Prepare a sheet of coarse paper by rubbing it with a dry mixture of sodium ferrocyanide and ferric ammonium sulphate. When you write on this with water the writing will be blue.

The American Flag

Make an outline drawing of our flag in hard lead pencil. Paint in the stripes with sodium sulphocyanide solution, the sky with potassium ferrocyanide solution, and the staff with tannic acid solution.

Using an atomizer spray the flag with ferric chloride solution and the proper colors will appear.

Fire Raisins

Pour over raisins one tablespoon of alcohol (ethyl). The idea which you wish to give your audience is that you are a fire eater and that you are eating burning raisins. The raisins coated with burning alcohol are picked up with a fork. The slight flame on

each raisin may be put out by expelling your breath as you place the raisin in your mouth.

Chemical Fireworks

A purple vapor can be produced by dropping crystals of iodine in a heated flask.

The following mixture when burned produces a green light: 12 parts of barium nitrate, 6 parts of potassium chlorate, 3 parts of sulphur, 1 part of powdered shellac, 2 parts of mercurous chloride, and one part of powdered charcoal.

The following mixture when burned produces yellow fire: 6 parts of potassium chlorate, 2 parts of sodium oxalate, 2 parts of sulphur, and one part of powdered shellac.

Silver fire can be produced by sprinkling a few grains of silver nitrate on glowing charcoal. Silver stars will be thrown off and the surface of the charcoal will be coated with silver.

The following mixture will produce red fire when burned: 40 parts of strontium nitrate or lithium nitrate, 13 parts of sulphur, 5 parts of potassium chlorate, and 4 parts of antimony trisulphide.

Greek Fire is produced (supposedly) by burning a mixture of saltpetre, sulphur, and pitch.

Blue Stars: 8 parts of potassium chlorate, 4 parts of copper sulphide, 2 parts of copper oxide, 4 parts of sulphur, 2 parts of mercurous chloride, and 1 part of powdered charcoal.

Pharaoh's Serpents: Make cones of a paste of mercuric sulphocyanide. Mercuric sulphocyanide can be made by mixing mercuric nitrate solution with potassium sulphocyanide solution and filtering off the white precipitate of mercuric sulphocyanide.

Rainbow Water

Prepare solutions of ferric chloride, potassium sulphocyanide, and potassium ferrocyanide. Wet a piece of cloth with the ferric chloride solution. Then dip this cloth into the potassium sulphocyanide solution and it will turn red. Finally dip the cloth in the potassium ferrocyanide solution and the cloth will turn blue.

Wine to Ink

The wine is prepared by mixing ferric chloride with an appropriate amount of potassium sulphocyanide solution. If this wine is poured into a glass containing a small amount of tannic acid the wine will be changed to ink.

X-Ray Eye

Have a typed message placed in an envelope. The message can be read by pouring ether on the envelope. This makes the envelope transparent momentarily so the message can be read.

Wine to Water to Wine

Wine is prepared by diluting tincture of iodine with water. When this wine is poured into a glass containing a small amount of a saturated solution of sodium sulphite solution the wine is apparently changed to water. If this water is poured into a second glass containing a small amount of red food coloring the water is apparently changed back into wine.

Fluorescent Writing

If you write on white paper using quinine sulphate solution as your ink, the writing will appear blue in the light produced by an ordinary Geissler Tube.

Mysterious Portrait

Write on glass with hydrofluoric acid. Allow to stand 5 minutes. Wash with water and dry carefully with a soft cloth. If you breathe on the glass the writing will appear.

Phosphine Smoke Rings

Set up a flask fitted with a one hole rubber stopper and a delivery tube made entirely of glass like the one used in the experiment for the preparation of oxygen. Place in the flask several small pieces of white phosphorus, 100 cc. of a strong solution of potassium hydroxide and a tablespoon of ether. If the flask is heated, phosphine will be formed as this gas escapes from the end of the delivery tube, just under water in a pneumatic trough. The instant the phosphine strikes the air it catches fire with the formation of beautiful smoke rings.

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THE ALGEBRAIC IDENTITY—ITS PLACE IN A FIRST COURSE IN ALGEBRA

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Most modern textbooks on Algebra stress the function concept and the idea of functional thinking as the basis for the unification of the subject matter. Important as this concept is, it usually fails to interest the average student and does not serve as the best means of transition from his previous work in arithmetic.

It is proposed here to show how the subject of algebra may be introduced in another way. We may well designate this procedure as the study of algebraic "synonyms" or, identities. The advantage of this method, as we shall see, is that it has close and definite connections with the mathematics with which the student is already familiar, and at the same time it will arouse his interest and curiosity in algebra. The treatment brings out more naturally how algebra is an extension and generalization of arithmetic—one of the ideas every teacher is anxious to impart to the minds of his students. It also furnishes an excellent means for motivating a considerable amount of the technique which is usually taught in a first course.

A number of examples will be given to suggest the type of material that may be used profitably during the first few weeks of a beginning course. It is not intended that any of these identities be proved at the outset, but they should be used to verify certain relationships between numbers, particularly, the positive integers.

Example 1. Let each student select two numbers, find half their sum and half their difference, and add these results together. He will discover, probably to his surprise, that the answer is always the larger of the two numbers. This is of course an application of the identity

$$\frac{a+b}{2}+\frac{a-b}{2}=a.$$

The teacher must call attention to the fact that this statement expresses in one stroke a truth concerning an indefinite number of problems of arithmetic.

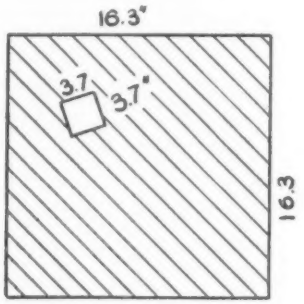
Example 2. The student may find the result of $8\times97\frac{1}{3}$

added to $8\times2\frac{2}{3}$ by multiplying 8 by 100. This is an application of the identity

$$ab+ac=a(b+c).$$

Example 3. An even more useful illustration of an algebraic identity is the one which tells us that the difference of the squares of any two numbers may be found by multiplying the sum of the numbers by the difference of the numbers, or

$$a^2-b^2=(a+b)(a-b).$$



Thus, if we are required to find the area of the shaded portion of the accompanying figure, we would ordinarily do this by computing $(16.3)^2-(3.7)^2$. However, the calculation is made much easier if, by means of the preceding identity, we write the area in the form

$$(16.3+3.7)(16.3-3.7)$$

or 20×12.6 which equals 252 square inches.

This particular identity is just as useful and interesting in the reversed form. For example, if we wish to multiply 83 by 77, we write these numbers as $80+3$ and $80-3$, so that by the preceding identity, the result is 80^2-3^2 which equals $6400-9$ or 6391. Such problems should be carried out without pencil or paper for practice in mental arithmetic.

Example 4. The identity

$$(2x+1)^2=4x(x+1)+1$$

shows that the square of every odd number when divided by 8 always leaves a remainder of 1. For, when x is an integer, either x or $(x+1)$ is even, so that $4x(x+1)$ is always divisible by 8.

Example 5. The next identity permits us to square mentally any number ending in 5. For a two digit number ending in 5, the

identity is

$$(10a+5)^2=100a^2+100a+25=100a(a+1)+25.$$

From this it follows that the square of 65, for example, is found by multiplying 6 by 7 and annexing 25. This identity also shows that the square of a number ending in 5 has an even digit just before the 25, since $a(a+1)$ is always an even integer.

Example 6. The obvious identity

$$100a+10b+c-(100c+10b+a)=99(a-c)$$

shows that if we subtract from any three digit number the number formed by reversing the digits, the remainder is always divisible by 99. This may of course be extended to numbers of more than three digits.

Example 7. An interesting identity, which may be called the Pythagorean identity, is

$$(m^2-n^2)^2+(2mn)^2=(m^2+n^2)^2.$$

Since by this identity m^2-n^2, $2mn$ and m^2+n^2 form the sides of a right-angled triangle, we have here a means of writing down an infinite number of right-angled triangles with integral sides, by merely assigning integral values to m and n. Thus, when $m=10$ and $n=3$, we obtain 91, 60 and 109. It is interesting to note, although we shall not prove it here, that the product $(m^2-n^2)2mn(m^2+n^2)$ is always divisible by 60.

Example 8. We shall now give an identity which enables us to perform mental multiplications with numbers beyond those usually memorized in the multiplication tables.

A special case will be considered first. To multiply two "teen" numbers, such as 18×13, we merely add 3 to the 18 obtaining 21, and after annexing a cipher to this result, we increase this number by the product of the units 8×3; we then find that

$$18\times13=(18+3)\times10+8\times3=210+24=234.$$

The identity underlying this method of multiplication is

$$(10+a)(10+b)=(\overline{10+a}+b)10+ab.$$

This identity permits us to multiply mentally numbers up to 20 by 20.

Example 9. A curious method of finger multiplication still used by certain peasants for multiplying numbers between 5

and 10, requires a knowledge of the multiplication tables up to 5 only, and may be explained on the basis of an algebraic identity.

To multiply 9 by 7, we raise four fingers (the excess of 9 over 5) of one hand, and two fingers (the excess of 7 over 5) of the other hand. The sum of the fingers raised, namely 6, is the tens digit of the result, while the product of the fingers down, 1×3, is the units digit of the result. This is a special case of the identity

$$xy = 10[(x-5)+(y-5)]+(10-x)(10-y).$$

Example 10. The common tests for divisibility by either 3 or 9, may be explained by an identity which is based upon our system of writing numbers. For the sake of concreteness, we shall consider the case of a four digit number.

Let N be the number and let a, b, c, d be the thousands, hundreds, tens and units digits, respectively. Then

$$N = 1000a+100b+10c+d.$$

The identity we wish to consider is

$$1000a+100b+10c+d = 999a+99b+9c+(a+b+c+d).$$

The first consequence of this identity is: (1), Every number may be written as 9 times an integer, plus the sum of its digits. Or, to put this in another way, (2), If the sum of the digits of any number is subtracted from the number, the result is always divisible by 9.

The following puzzle is based on this last fact. Write down any four digit number, subtract the sum of the digits from this number. If any three digits of the result are given, the fourth one may be determined. (It is clear that if 0 or 9 were to be the digit omitted, there would be an ambiguity.)

Another consequence is that if a number is divisible by 3 or 9, the sum of the digits is divisible by 3 or 9; and conversely, if the sum of the digits is divisible by 3 or 9, the number is also divisible by 3 or 9, as the case may be.

For, by (1) above, we have $N = 9k+S$, where k is some integer and S is the sum of the digits of N. Hence, $N/9 =$ an integer $+ S/9$.

Another application of the above identity, leads us to the familiar method of testing the results of the elementary operations by casting out 9's. For example, if we were to test the accuracy of a multiplication, we make use of the fact that the

remainder of the product of two numbers, after casting out 9's, is the product of the remainders. This is easily seen by writing $M=9k+r$, $N=9l+s$, and $M\times N=9n+rs$.

Example 11. The identity to be considered next gives us the basis for the validity of testing for divisibility by 11. The identity is

$$1000a+100b+10c+d=1001a+99b+11c+(-a+b-c+d).$$

From this we see that

A number is divisible by 11 if and only if, the sum of the digits in the odd numbered places minus the sum of the digits in the even numbered places (counting from right to left) is so divisible.

To carry out the divisibility tests for both 9 and 11 for a number containing any number of digits, one would have to make use of the criteria of factorability of 10^n-1 and 10^n+1 by 9 and 11. These identities are usually given for $n=1, 2, 3, 4, 5$.

Example 12. A seemingly trivial but interesting identity is

$$MN=a(Nx^3)+b(Nx^2)+c(Nx)+dN, \text{ where } M=ax^3+bx^2+cx+d.$$

If the number M is expressed in the scale of 2 $(x=2)$, this identity leads to a method of multiplication that consists of halving and doubling. Thus, to multiply 17 by 65, we proceed as follows:

	17		$65=65\cdot 1$
Dividing by 2 and dropping remainders	8	Multiplying by 2	$130=65\cdot 2$
	4		$260=65\cdot 2^2$
	2		$520=65\cdot 2^3$
	1		$1040=65\cdot 2^4$

Adding those numbers in the second column which are opposite the odd numbers in the first column, namely, 65 and 1040, we obtain 1105, the required product.

To see how the preceding identity is applied, we write 17 in the scale of 2, and obtain

$$17=1\times 2^4+0\times 2^3+0\times 2^2+0\times 2+1$$

so that $17\times 65=1(65\cdot 2^4)+0(65\cdot 2^3)+0(65\cdot 2^2)+0(65\cdot 2)+1\cdot 65$ and we notice that only the first and last terms on the right contribute anything to the result.

Example 13. This identity may well be introduced by citing

the following cases:

$$1+2 = 3=2^2-1$$
$$1+2+2^2= 7=2^3-1$$
$$1+2+2^2+2^3=15=2^4-1$$

all of which are special instances of the identity

$$1+2+2^2+2^3+\cdots+2^n=2^{n+1}-1.$$

For, if we let $s=1+2+\cdots+2^n$, then $2s=2+2^2+\cdots+2^n+2^{n+1}=2^{n+1}-1+s$, so that $s=2^{n+1}-1$.

The familiar problems of finding the sum of money at the end of thirty days, if each day's saving is twice that of the preceding day; or, the number of grains of wheat on a chess board if each square contains twice the number of grains as the preceding one, are all solved by this identity.

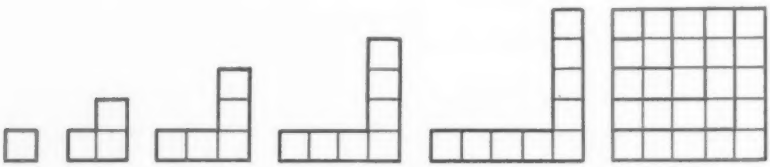
Example 14. An identity which permits us to add all consecutive whole numbers from 1 to any number however large, should be of interest to the beginning student. Suppose that we wish to add 1, 2, 3, 4, . . . , up to 1999. We notice that 1 and 1999 add up to 2000, that 2 and 1998 add up to 2000, that 3 and 1997 add up to 2000, etc., and since 1000 such pairs may be formed, the entire sum amounts to 2000×1000 or 2,000,000. This is a special instance of the identity.

$$1+2+3+\cdots+n=(n+1)n/2.$$

Example 15. An identity related to the preceding one is that the sum of the first n consecutive odd numbers is the square of n, that is,

$$1+3+5+\cdots+(2n-1)=n^2.$$

The proof of this identity may be given most readily by diagrams. Let us represent each unit by a square. Then it is clear that we may fit each figure beginning at the left into its first neighbor at the right, and finally obtain the square at the extreme right. The diagrams are drawn for $n=5$.



Example 16. The identity which enables us to add the squares of the consecutive integers beginning with 1 is often found very useful. While the proof of such an identity may also be explained diagrammatically, we give a derivation which is interesting in that it is applicable not only to the squares but to higher powers as well. This derivation is based upon the simple identity.

$$x^3-(x-1)^3=3x^2-3x+1$$

When
$$\begin{aligned} x&=1, \quad 1^3-0^3 &&=3\cdot 1^2-3\cdot 1+1\\ x&=2, \quad 2^3-1^3 &&=3\cdot 2^2-3\cdot 2+1\\ x&=3, \quad 3^3-2^3 &&=3\cdot 3^2-3\cdot 3+1\\ &\cdots\cdots\cdots\cdots\\ x&=n, \quad n^3-(n-1)^3 &&=3\cdot n^2-3\cdot n+1. \end{aligned}$$

Adding these expressions for the values of x from 1 to n, we see that the left member reduces to n^3 since all the other terms cancel each other. Hence,

$$3(1^2+2^2+\cdots+n^2)-3(1+2+\cdots+n)+n=n^3.$$

Using the identity in Example 14 for the second parentheses on the left, we find by a simple calculation that

$$1^2+2^2+3^2+\cdots+n^2=n(n+1)(2n+1)/6.$$

Attention should be called to the fact that, while certain of the identities given are true for any numbers, others are true for positive integers only.

PREDICT END OF TOOTHACHES AND DENTISTS BY YEAR 2040

The year 2040 may see the end of toothaches and of the type of dentistry practised today, if American dentists continue the rate of progress during the next 100 years that they have made in the past century.

This cheerful prospect for future Americans was pictured by Dr. John E. Gurley, of San Francisco, at the Baltimore Dental Centenary, though Dr. Gurley drew the picture in the more conservative phrases of a scientist.

"The fundamental requirement of a profession," he said, "is that it shall make the need for its ministrations unnecessary." In other words, it must put itself out of business.

"Thinking in terms of dentistry's future ministrations," he continued, "it is no wild guess to suggest that we will go swiftly into the field of prevention and that ere many years hall have passed, there will be witnessed a great reduction in caries (tooth decay). Dental practice, and I may suggest medical practice, will change from curative and reparative to health service."

PROJECT-TECHNIQUES RELATIVE TO EXHIBITIONS

MAITLAND P. SIMMONS
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Never before in the history of the teaching of general science has there been such an urgent need for teachers to make classroom work more interesting and challenging to all concerned.

Surely, the educator of today must be interested in making the subject-matter more attractive and utilitarian to the pupil. One method of approach, then, would be to encourage pupil participation in the construction of scientific projects relative to units studied. Since it is quite generally recognized that exhibitions create a school interest, the primary purpose of this paper, therefore, will be to present definite fundamental techniques that were employed in the development of varied projects for the New Jersey and New York[1] Science Fairs, New Jersey Science Teachers' Association Meetings, and our annual school library exhibits.

From long teaching experience in ninth-grade general science, the writer finds the following techniques quite successful for the development of general science exhibits:

1. It appears desirable to focus the attention at the beginning of the semester, since several months are required for the best results. Sufficient time must be allowed to avoid rush and confusion.

2. The creation of a desire for this work is essential. Of course, a certain amount of this latent interest is present with some pupils. Therefore, the teacher, always mindful of this, needs only to change this interest into whole-hearted, purposeful activity. Usually, the remaining pupils develop some degree of enthusiasm and congeniality through coöperation. Interest is then stimulated by showing a few worthwhile projects made by former pupils and by the teacher. Newspaper clippings of previous years, as well as magazine articles with pictures of pupils and their models,[2] particularly those relating to our school, are shown to the class. It is always well to have some of the better work photographed. Newspaper photographers will

[1] The American Institute Science and Engineering Fair, New York City.

[2] Simmons, Maitland P., "Their First Scientific Project." *Science Education* 22: 310–311; November, 1938.

Simmons, Maitland P., "Science for Leisure Time." SCHOOL SCIENCE AND MATHEMATICS 39: 623–627; October, 1939.

gladly do this, as most newspapers welcome school news. Young people benefit by this kind of publicity. Next, mention that if the project is well done, it will be exhibited at the school library, a science fair, a museum, or some public place where pupils, parents, friends, and faculty may acknowledge their efforts. Another excellent means of motivation is to mention the presentation of awards by Science Fairs[3] and our school. Pictured below is a project that received a third prize at The American Institute Science and Engineering Fair. These awards

William Lesser, Irvington High School, checking on the construction of his model, the George Washington Memorial Bridge.

are given out during an assembly program; certificates of merit, printed by the school, are presented at another auditorium period to those less fortunate in receiving one from these Science Fairs. Instead of building models, it might be advisable to suggest that some can make attractive booklets and posters as an added feature for the school library exhibit. To guide the pupils, several scrapbooks and drawings are usually shown to the class.

3. Projects that are workable, unusual,[4] and varied should be

[3] Six New Jersey Science Fair awards in 1936 and ten in 1937 were presented to members of the general science classes of Irvington High School. The Fair was discontinued in 1938.

[4] Simmons, Maitland P., "A Bridge on Exhibition." *Science Education* 24:43; January, 1940.

suggested. Stress originality;[5] encourage projects that have not been displayed in previous years. A list is posted on the bulletin board. Interested pupils sign their names, and thus feel a strong sense of responsibility for a completed task. Often group projects may be proposed, since they act as a tonic stimulating coöperation and effort. For a project to be workable, the materials should be inexpensive and easily obtained. Discarded bits of apparatus and broken toys classified as "junk" may be frequently used. In many instances, materials may be procured from school manual training rooms, industrial concerns, or pupils' homes where the work is done during leisure hours. The construction should be simple, material light in weight yet strong enough to withstand transportation. Most important is that it should involve a challenge to the understanding and the reasoning ability of the individual. It is especially significant that whenever possible, the pupil should be given an opportunity to select, carefully plan, and execute the entire project.

4. Dimensions for construction can now be given. The standard size for competitive exhibits is usually two (2) feet square and not over three (3) feet high. This makes it convenient to handle, and at the same time shows up clearly. The base, generally plywood, should be neat in appearance, as that gives an attractive setting. Objects should be large and brilliantly colored. Pupils actually participating should be given clear-cut pictures, free from confusing details, as guides for their creative study. These illustrations may be obtained from up-to-date science textbooks, newspapers, and handicraft magazines. Suggest libraries, hobby shops, industries, museums, expositions, and some department stores for the pupils to visit, as the displays in those places will give some ideas as to construction and how the piece of work can be exhibited advantageously.

5. Definite check-ups of the progress must be made. These may take the form of periodic conferences during class period while other pupils are working on that day's lesson assignment. It is highly important to observe that every detail in the picture is being carried out. From these individual discussions the teacher will discover that some will be unable to complete their work because of illness or the acceptance of a part-time job. Of course, from these informal talks the teacher has an excellent opportunity to advise and counsel pupils in solving various common school problems.

[5] Simmons, Maitland P., "Originality of Scientific Models." *Science Education* 22. 195; April, 1938.

6. The teacher should request the submission of the projects several weeks before the date of exhibition. Stress caution when bringing them to school, as they are easily broken if not handled carefully. When a well-done project comes in, display it to the class in order to motivate the students. The others will thus hasten to complete their own so that theirs may be shown to the classes. When the projects are finished, select the best ones for the Science Fairs; reserve the others for the school exhibition. Each pupil should make a large explanatory poster for the background of the project. An attractive poster from a previous year will serve as an excellent guide. At least two weeks should be allowed for this work. Here are a few salient points relative to successful poster-making:

a. Obtain durable material such as oak tag.

b. Show contrast in coloring.

c. Use few, plain, large letters, figures, and characters.

d. Have ample space between right and left margins, and between lines.

e. Make the significant ideas stand out, by omitting unimportant details.

f. Use correct spelling.

7. As the projects are now ready for exhibition, the following essentials are listed:

a. Several of the larger boys are needed to move the heavier pieces. For transportation, projects must be packed with the utmost care and protected from rain. Be sure to include setting-up materials.

b. A few girls can decorate the tables and arrange the materials attractively. School desk blotters may be used effectively as a base for each exhibit. The teacher should see that all projects are skillfully displayed.

c. Two reliable girls may take charge of the exhibition. Their duties would be to keep objects arranged in an orderly fashion, to prevent breakage and loss of materials, and to answer questions visitors may ask.

It is interesting indeed to note the keenness of attention and the eagerness that pupils put forth in their piece of work. A continuity of interest, so often lacking in day-to-day teaching, prevails, as pupils are building other projects relative to various academic subjects.

This paper is presented in the hope that other teachers may be stimulated to make their class work more "interest-creating"

and vitalizing to the individual. The typical classroom tendency is to become too abstract. Real objects or models give definite meaning to many words and thereby combat verbalism, or the use of words without attention to thought. No instructor can expect to reach the highest degree of efficiency unless he uses real life situations in the classroom. Project teaching is generally more realistic than traditional teaching.

SUMMER GRADUATE INSTITUTE OF ARMOUR INSTITUTE OF TECHNOLOGY

Armour Institute of Technology, Chicago, according to the announcement made by Dr. L. E. Grinter, vice-president and dean of the graduate division, will conduct a three-term SUMMER GRADUATE INSTITUTE for engineers, professional men, industrialists, and educators in engineering and science beginning with the summer of 1940. It is planned, according to Dr. Grinter, who is in charge of the Summer Institute beginning with this year, to invite scientists of great distinction to lecture each summer on modern developments in engineering and science.

The faculty of the SUMMER INSTITUTE for 1940 includes, in addition to prominent men from the staff of Armour Institute, the following visiting professors: Ralph M. Barnes, Iowa State University; John R. Baylis, City of Chicago; Barnett F. Dodge, Yale University; William R. Everitt, Ohio State University; C. C. Furnas, Yale University; E. Hurwitz, Chicago Sanitary District; Lydik S. Jacobsen, Stanford University; G. B. Karelitz, Columbia University; Gabriel Kron, General Electric Company; Charles O. Mackey, Cornell University; Floyd W. Mohlman, Chicago Sanitary District; J. C. Morrell, Universal Oil Products; H. C. Taylor, Western Electric Company; Stephen P. Timoshenko, Stanford University; Frank C. Vilbrandt, Virginia Polytechnic Institute; and John I. Yellott, Stevens Institute of Technology (Mr. Yellott in September 1940 will become Head of Mechanical Engineering at Armour College of Engineering of Illinois Institute of Technology).

The faculty members of Armour Institute who will cooperate in this summer program include: Paul L. Copeland, Lloyd H. Donnell, H. P. Dutton, Rolf Eliassen, Lester R. Ford, L. E. Grinter, Vasili I. Komarewsky, Max Jacob, Harry MacCormack, Joseph Marin, Myril B. Reed, Hans Reissner, Max Sadowsky.

This SUMMER INSTITUTE is divided into seven separate and distinct divisions and the program of specialized classes have been arranged to provide an opportunity for graduate work on the highest possible plane. These divisions include, Advanced Mechanics, Chemical Engineering and Chemistry, Civil and Sanitary Engineering, Electrical Engineering and Physics, Mechanical Engineering, Industrial Engineering and Applied Mathematics. The first term of the Summer Institute will extend from June 17 to July 13; the second term from July 15 to August 10; and the third term from August 12 to September 7.

It was also announced that the SUMMER GRADUATE INSTITUTE will be conducted in future years under the direction of Armour College of Engineering of Illinois Institute of Technology. This will result from the merger of Armour and Lewis Institute, expected to become effective as of September 1940, and the subsequent changing of the name of the new combined colleges to Illinois Institute of Technology.

THE PLACE OF NATURE STUDY IN THE PUBLIC EDUCATIONAL SYSTEM

RICHARD GERSTELL
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The various agencies responsible for the administration of the natural resources of the several States of the Union are today beset by numerous problems of major importance. Not least among these is that pertaining to education.

Though many educational programs conducted by state conservation departments have been only partially successful, or have failed, because they were based upon false premises, or as the result of ill-chosen techniques, or for other similar reasons, it would appear that the great majority of all such plans contain one, single, basic weakness to which their usually limited degree of success may well be attributed. This is the fact that *practically every attempt at state conservation education has involved overestimation of the fundamental knowledge of the natural phenomena possessed by those persons whom it was designed to reach.*

While it is the duty of the state organization to outline and encourage public participation in sound programs aimed at the conservation, or wise use, of natural resources, the responsibility for teaching the biological principles upon which such plans may be founded rests upon those in charge of the various school systems. Thus, the purpose of this discussion is to show the place of nature study in the public educational system by pointing out just how such instruction will better enable the state conservation departments to develop active, statewide cooperation in sound conservation programs.

As an administrative officer in a state game department, the writer will utilize for illustrative purpose a major wildlife problem currently faced by Pennsylvania, Michigan and numerous other states. This relates to the proper management of the deer herds within those governmental units.

During the first few years of the twentieth century, Pennsylvania was a state whose natural resources lay sadly despoiled. Practically all game was shot out; the mountains had been stripped of their forests; industrial waste had polluted the majority of streams and rivers. Fortunately, however, a state game commission had been created by act of legislature in the late 1890's. It is true that the organization was then exceed-

ingly small and pitifully underfinanced, but it was under the direction of a few extremely active and highly capable men. Principal among the early aims of the group was the preservation of the whitetailed deer. This species had once been abundant throughout the state, but following years of market hunting and other forms of constant persecution, it was in 1900 practically extirpated.

The very size of the conservation organization, not to mention lack of funds or the unreceptive state of the public mind, rendered any attempt at educational measures designed to increase the deer herd entirely out of the question. Thus, the Commission strove to accomplish its purpose largely through the enactment of protective legislation. These efforts continually met with stubborn opposition from all sides.

One of the early legal restrictions desired by the department was a law which would temporarily provide protection for female deer. It was felt that this would allow for a greatly needed increase in the breeding stock.

The request was met by an unusual storm of public protest. A statewide organization of hunters quickly banded together for the purpose of preventing the approval of such legislation. After a long struggle the act was finally passed. This was done, however, not as a means of increasing the deer, but because it was felt that it would lend to the protection of human life by "making the hunters look for horns" before shooting.

This so-called "buck law," along with a few other sound but simple measures, combined with the natural forest development, then taking place largely as a result of protective action, to produce by the early 1920's a deer population increased to a point where it exceeded the carrying capacity of the range upon which it had to depend for support.

Some years before, the Commission had foreseen the need for reducing the herd through the removal of some of the surplus females, but the mere mention of action designed to accomplish such a thinning met with even greater protest than had the request for legal protection of the does. This was due to the fact that practically all interested persons attributed the vastly enlarged size of the herd not to the peculiarly favorable combination of conditions which had actually made such growth possible, but solely to the protection of the females of the species.

At the present time, due largely to the pitiful sight of thou-

sands upon thousands of winter-killed deer, to the results of a constant educational program designed to point out the causes and effects of over-browsing of the range, and to the fact that a number of open seasons for the killing of antlerless deer have come and gone without resulting in the annihilation of the herd as many persons had prophesied, it has finally been possible to place the herd under reasonable management.

During that fifteen year period from 1923 to 1938, wherein sentiment which has finally allowed for better management of the herd was built up, both the deer and the range suffered irreparable damage. Also, the constant bickering, the law suits designed to prevent the opening of seasons for the killing of female deer and similar disputes, have tended to prevent a unanimity of opinion on conservation matters.

So much for the history of the problem. The question is: Just how might nature study education have tended to prevent its development or have aided in its solution? In the first place, had the public possessed even a rudimentary knowledge of the laws of nature, it is probable that the temporary protection of the females might have been plead for rather than fought against. Secondly, had the hunters gained some knowledge of the life history and ecology of the whitetails, they doubtless would have realized that the rapid increase of deer which took place during that decade extending roughly from 1913 to 1923 was the result of an unusually favorable combination of conditions, rather than of the protection of the females alone. Finally, and most important, had the majority of interested persons gained through early education some knowledge of the basic principles of biology, it might have been possible to have applied the necessary corrective measures at an earlier date, thus preventing much of the damage suffered in recent years.

The point last mentioned is worthy of more detailed consideration. In the early 1920's, when the deer problem first became glaringly apparent, the Commission began a vast educational program designed to "sell" the need for herd reduction. In this work lectures, motion pictures, printed material of various types and radio broadcasts were utilized. Time and time again attempts were made to "drive home" the simple fact that the deer problem was primarily the result of a shortage of winter food; that the nutritive requirements of a greatly increased herd had exceeded the food producing capabilities of the range. Aided to no small extent by the publicity given the suf-

ferance of heavy winter losses caused by the excessive demands placed on the range, the educational program slowly, but very slowly, began to bring the desired results. Here the important question is: Why did the program not move forward more rapidly?

It would now appear that again in this case, even though based on sound principles and utilizing proper techniques, the effectiveness of the program was limited because it assumed on the part of the hunters a knowledge of deer which they did not possess. It seems that practically all interested persons failed to grasp and accept the fact that there was a shortage of winter food *primarily because they did not know what naturally constitutes the normal winter food of deer*. Few hunters realized that the principal source of winter nourishment lies in the tender twig tips of numerous woody plants. The knowledge of the majority concerning this matter is probably typified by the reply of one "experienced" hunter who was asked upon what deer depend for winter food and who replied, "Oh, grass and things, I guess."

Along somewhat the same lines, though possibly even more surprising, was the discovery during recent open seasons for the killing of antlerless deer, that only very few of the thousands of hunters who legally shot fawns born the previous summer recognized them as such. These animals, weighing from thirty-five to seventy pounds hog-dressed, were in most instances estimated to weigh from eighty-five to one hundred thirty pounds and classed as yearlings, or even mature animals. There seemed to be little or no knowledge of the shedding of a deer's coat, or of the time when a fawn loses its spots.

At least to the writer, these points would clearly seem to indicate that if we are to have active cooperation in statewide programs aimed at the conservation of natural resources, we must have a public which possesses at least an elementary knowledge of the natural sciences. Furthermore, it appears that nature study represents an essential part of any program designed to accomplish this educational aim. Thus, the need for a general nature study program in the public educational system becomes obvious.

It will indeed be interesting to see how many of the things which astronomers have learned during this last century will have to be unlearned in the next.—REGINALD L. WATERFIELD, *A Hundred Years of Astronomy*.

NEW MATERIALS AND EQUIPMENT FOR BIOLOGY TEACHING*

JOSEPH W. RHODES
The Senior High School, Beloit, Wisconsin

Herbert Spencer was a firm believer in the supposition that early ideas are not usually true ideas. The undeveloped intellect, be it that of an individual or that of a race, forms conclusions which had to be revised before they reached a correspondence with realities. Were it otherwise, there would be no discovery, or no increase of intelligence. What we call the progress of knowledge, is the bringing of thoughts into harmony with things; and it implies that the first thoughts are wholly out of harmony with things or in very incomplete harmony with them.

To prove his point Spencer cited the primitive notions mankind had as to the structure of the heavens, the form of the earth, and more recently the composition of matter. To quote Spencer: "In all these cases, men set out with beliefs which, if not absolutely false, contained but small amounts of truth disguised by immense amounts of error." As man progressed he has discovered new facts usually after a period of experimentation in which he has disproved many possibilities. A study of the history of medicine will show in what ways traditions were a hindrance to advancement.

The progress that the medical profession, for example, has made during the last one hundred years may be attributed to the use of the scientific method of procedure. Other fields of endeavor soon realized the benefits derived from this scientific method. One may ask: "Why did the early botanists, zoologists, and anatomists progress so slowly?" The lack of scientific equipment was a fundamental cause, but I believe the lack of understanding and the superstitions of the average adult were the primary causes. Vesalius, often referred to as the "father of human anatomy," would risk his life taking the hanging skeletons from the scaffold under cover of darkness. The method of embalming used by the early Egyptians vividly portrays the influence of superstitions. The most expensive process of embalming, costing the equivalent of a thousand dollars, com-

* Read before the Biology Section of the Central Association of Science and Mathematics Teachers, November, 1939. Mimeographed lists of "New Materials and Equipment for Biology Teaching" were distributed to those present. These lists will be sent to biology teachers on request as long as the supply lasts. Address: Editor, SCHOOL SCIENCE AND MATHEMATICS, 7633 Calumet Ave., Chicago, Ill.

menced with an incision in the wall of the abdomen. The man who made the incision ran away as soon as the act was completed to avoid being stoned; this was a ceremonial gesture to indicate his offense against the dead. The embalmers then removed the abdominal viscera and preserved them in vases of clay, limestone, or alabaster. They next removed the brain through the nostrils by means of an iron hook. The cavities of the skull and abdomen were filled with spices. The body was next soaked for seventy days in a solution of salt, and afterward coated with gum, wrapped in cloth, placed in a wooden coffin, and then deposited in the burial chamber together with the jars containing the viscera.

It is obvious that the so-called civilized countries have progressed far beyond this type of practice. However, only last year a survey was made that revealed the fact that over 250 American newspapers carried astrological information. A good share of the reading public still desires that type of service. But the encouraging note is that there is an increasing number of papers and magazines that perform a real service in the presentation of scientific news, and their influence will no doubt spread. And as Dean MacQuigg of Ohio State comments, "Rather than delay science one moment, let us urge society to catch up with it. Help to ennoble and not degrade the uses of science to a better life of the future."

However, the biology teacher is not without a challenge—a challenge that has faced him along with the teachers of the other natural sciences. Unless the biology teachers find a way to improve, maintain, and expand the biological teaching, the teachers as well as society as a whole will suffer. And it has been said that biologists cannot forget that those developments which have been most conspicuous in breaking the intellectual traditions of the past lie within or near their own field of life science. From here has come much new evidence that life itself has evolved from the non-living, and that one living species is slowly transmuted into another, under natural law.

Definite changes have occurred in the methods used and subject matter taught. May I briefly describe a more-or-less typical biology course offered in a large high school thirty years ago. Biology was required in the first two years of each of the seven science curricula offered; first, as an introduction to science itself and second, as a foundation for later biological and pharmaceutical courses. In each course the fundamental principles

were emphasized but the special needs of the various groups, namely pharmacy, nursing, dietetics, etc. largely determined the choice of materials. No one would deny that a course of that type would be helpful to those who intended to follow those particular professions. But it is of far greater importance to arouse life interests which will stimulate the learner to continue his interests after his formal schooling has been completed. The emphasis should be placed on the needs of individuals. This change of emphasis is clearly portrayed in the statement of the Committee on the Function of Science in General Education, of the Progressive Education Association: "The purpose of general education is to meet the needs of individuals in the basic aspects of living in such a way as to promote the fullest possible realization of personal potentialities and the most effective participation in a democratic society."

Let us consider briefly some of the underlying changes that have brought about this shift of emphasis. Due to the enormous increase of enrollment in the high schools the classes, in many cases, have increased greatly in size prohibiting a daily evaluation of each student's work. In some instances other subjects have been crowded into the curriculum allowing less time for biology. The range of mental ability has increased with the all-inclusive enrollment, thereby placing a greater responsibility upon the teacher. A larger number of students are not book-minded and consequently would receive very little benefit from textbooks when used in the traditional academic manner. And lastly, supervised study is more commonly used because of a greater appreciation of adverse study conditions found in some homes.

However, a word of warning is offered by Prof. Fitzpatrick, of Columbia University, in our enthusiasm for mass education. He believes, and rightly so, that we should not forget the students who possess more than the average ability, and are motivated by more than normal interest. It must be remembered that it is in this group that many of our future leaders will undoubtedly be found.

In 1925 George W. Hunter made a survey of a national character in which he compared the phases of biology teaching with those of 1908. He found a very marked decrease in the amount of time given to taxonomy, morphology and ecology. Human physiology and natural history showed no appreciable change, but the phases in relation to man and human biology showed a

marked increase. There was less emphasis on the formal teaching procedures but greater effort was exerted by the teachers to make use of the children's interests and instincts. The progressive teacher will always look upon this practical activity as a continued discovery and the development of student potentialities. Paul Sears, of Oberlin College, believes that it is only the remarkable improvement in textbooks and visual aids that keeps us from doing worse than we do.

Let us make a definite comparison of the textbooks of today with those of the past. Vitamins are not mentioned in the older texts but are given an important place in the study of the diet and digestion in the present day text. Their importance cannot be questioned as one may conclude from the following verbal portrayals: Picture an emaciated maniac with swollen red tongue and scaly skin, dying a lingering death from pellagra, "the hard times disease of the South," restored in a few days to health and sanity by magic pills of nicotinic acid. Pellagra claims 400,000 victims in the United States each year, and of these about 8000 are fatalities. Imagine how a feeble paralytic, with failing heart and swollen, paralyzed legs, suffering from an advanced form of beri beri, the scourge of the Orient, can be made normal again in a marvelously short time by a man-made compound named thiamin, or vitamin B_1. Or, how a twitching, irritable, whining infant with spongy bleeding gums, and the hemorrhagic joints of scurvy is soon changed into a placid, smiling healthy-looking baby by ascorbic acid, vitamin C, the protective element in orange, lemon and tomato juices. Dr. John C. Goffin, of the Los Angeles Schools, estimates that from one third to one half of our population is vitamin starved.

The internal structure of life types such as the clam, earthworm, and crayfish is considered in detail in the older texts but is omitted or only briefly mentioned in the newer ones.

Insect control, which was only briefly considered, is now given a prominent place, probably because man spends more money fighting insects than any other form of life other than himself. The increasing importance of conservation is also noted by the space given it. It wasn't mentioned thirty years ago. The older texts thought little of insects and disease and of plant and animal breeding but the more recent texts give them the consideration they deserve. It is interesting to note the change of emphasis found in the study of the human foot. The older texts presents its anatomy in terms of the three types of levers, while

the more recent texts stress foot hygiene. Short biographies of famous biologists are now found in most of the newer texts. Other phases of biology recently given greater emphasis include history of man and tobacco and table beverages. The progress can be readily seen when comparing a recent text with one such as Herbert Spencer's, *Principles of Biology*, published in 1866, in which he devotes an entire chapter of twelve pages to the definition of life. May I state his conclusion: "Life is the definite combination of heterogeneous changes both simultaneous and successive."

The workbooks and laboratory manuals have closely followed the trends considered important in the textbooks. The various questions and exercises emphasize the phases of biology that are employed in the textbooks. Much less is given over to taxonomy, morphology and ecology.

Visual aids are not new to biology as they have been employed from the time the instructor brought in specimens for study. Visual aids should not be selected so much to prepare the student for college as to help him better understand the life about him. In a general survey of the motion pictures available for biology it was found that there were only a few available for plant classification. Many beautiful and interesting films may be had on plant and animal ecology. There are excellent pictures on heredity, plant and animal breeding and genetics, made mostly by educational producers. Sound pictures are generally considered better teaching aids because of the skillful editing, particularly if it is done by educational producers. The biology motion picture material is plentiful, generally well done technically, but sometimes not adequately adapted to the curriculum needs.

High school biology is particularly fortunate in having access to an abundant supply of new books of the general reading and the reference types. The general reading type of book should receive greater emphasis as it often serves as a means of motivation and appreciation of life about us. Some of these books are composed largely of unusual pictures of life forms.

The various supply companies have followed closely the desires of the teachers. This is in evidence when attention is given to the newer items found in their catalogs. Live materials, such as observation beehives, ant nests, salt water aquaria, fresh water and balanced aquaria, rats and mice for experimental purposes, are considered of greater importance. Additions are con-

tinually being made to an already adequate variety of charts and models.

And now briefly, what about the future? What can we look for? College preparatory courses will cease to exist as each biology course will be taught as an end in itself. The student will be taught science as a means to help solve his social problems, to help society catch up with science before the gap becomes too great, and before science becomes an instrument of destruction instead of a means of human betterment. Visual aids will become more important as a teaching aid, and good general reading books and magazines will help to bring about an appreciation for the forms of life in the student's environment that will carry over to a fuller and happier life afterward.

CHEMISTRY COURSES FOR TEACHERS AT WESTERN RESERVE UNIVERSITY, SUMMER SESSION, 1940

The Department of Chemistry of Western Reserve University, is offering in the summer session for 1940, beginning June 17, in cooperation with the Division of Chemical Education of the American Chemical Society, given by Professor Rufus D. Reed of New Jersey State Teachers College, two special graduate courses, looking to the solution of two important problems of teaching chemistry in secondary schools.

The first course, called "Work Shop in Chemistry" attacks the important problem of the minimum and optimum essentials in high school chemistry, which the average normal pupil can accomplish.

The second course has in mind the ultra-bright student, who chafes at the slow pace of the average student and demands and should have extra curricular chemical activity. This course styled, "Demonstrations and Projects in High School Chemistry" is a seminar course to develop experiments and projects suitable for extra laboratory work for the brighter student and for science programs and assemblies.

These courses are offered in the afternoons, giving plenty of time for study and for observation in the various summer high school sessions in chemistry in Cleveland in the morning.

Arrangements will be made for visiting teachers to visit various schools in operation in Cleveland and other points of interest.

Teachers of chemistry, who take these courses, will have the opportunity to attend a series of sixteen lectures on current national and international issues in the Summer Institute on Current Affairs. The lecture series will have as its central theme, "The Major Issues of 1940." The lecturers invited to participate in the series are chosen because of noteworthy contributions to the problems.

A course of lectures on topics of special interest in Organic Chemistry are offered at a time not interfering with the above courses. In addition, General Chemistry and Physical Chemistry, Qualitative and Quantitative Analyses, are offered. By attending both the regular and the post session, it is possible to obtain a year of General Chemistry and Qualitative Analysis, or a year of Organic Chemistry, in ten weeks.

For further information on these courses and other courses offered in the Department of Chemistry, write the Summer Session, Western Reserve University, Cleveland, Ohio.

HUMANIZING THE CURRICULUM OF THE NATURAL SCIENCES AND MATHEMATICS[1]

J. S. GEORGES
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Educational objectives, however defined, are dependent upon the type of education and educational principles employed. In the final analysis, the merit of any educational curriculum must be evaluated in terms of the objectives which it proposes to attain and whether or not they are actually attained. Now it is quite reasonable to assume that an educational institution may set up any type of curriculum it desires and if it can sell the idea to enough students it may undertake to attain its educational objectives by the best feasible methods. However, when that institution or educational system seeks public support because it proposes in turn to serve the society educationally, then the society has the right to inquire whether or not the proposed objectives of the institution are desirable, and if found desirable, whether or not they are attainable, and if attainable, whether or not they are worth the effort and the expense.

An educational system which is created and supported by society must justify its existence and maintenance by meeting the social needs prescribed by society. Obviously as the social order undergoes natural changes and transformations, the social needs, which must be met by the educative process, must of necessity also undergo a parallel transformation. The type of education most desirable in any particular social order and at any particular period of time is that which meets the social needs of that order at that time.

The present complex life of an individual member of society and of the society itself is prescribed by the type of civilization in which the individual and the society are living. The society is chiefly concerned in preserving that civilization. It can do so only if the individual members, which constitute the social order, have been trained by the educational system to make the necessary social adjustments through proper attitudes, proper conduct, and proper living. It seems but reasonable that this should constitute the primary aim of education, and if this primary aim of education is to be realized, the educational system must see to it that every subject which finds a place on its curriculum contributes materially to its attainment.

[1] Presented at the University of Illinois, November 3, 1939.

For the purposes of instruction, the social needs, however determined and defined, become the learning products of the various specific subjects of the curriculum. But a subject such as physics or mathematics is a body of concepts, principles, processes, and facts which deal with definite phases of man's attempt to understand the world about him and to adjust his life in conformity with such an understanding. Consequently, at the very outset we are faced with this important problem. Shall the instructional program of physics or mathematics be concerned primarily with the specialized training of certain individuals to become physicists or mathematicians in order to understand adequately the concepts, processes, and principles of physics or mathematics, or shall this training be non-specialized in character by making accessible to all as many of the concepts, processes, and principles of these subjects as the individual is able to grasp under proper guidance? In other words, can the natural sciences offer other then specialized courses? And if such courses are possible and are teachable, can they serve to attain the social needs of the individual as well or better than the specialized courses?

In the organization of a course of instruction, just as in the organization of an educational system, it is essential to come to some sort of agreement as to the nature and significance of the social needs of the individual. Also it is equally essential to determine subjectively or objectively what type of education is the most effective in the attainment of the social needs we propose.

In the excitement and confusion which have accompanied our efforts to determine the social needs as well as the instructional methods to be employed in ther attainment we have either focused our attention and energies upon the individual and his needs, calling his needs the desired social needs, at the same time have neglected to ascertain the needs of the social group; or else we have gone to the other extreme by setting up certain patterns of procedure which will tend to realize the desirable outcomes for the betterment of the social order, and have undertaken to make the individual fit that standard pattern. In both cases we have fallen short of realizing the most important facts and fundamental principles in the utilization of the educational values of the various subjects. We have set up artificial measuring rods by means of which a given subject is weighed and evaluated and either found wanting and discarded or found worthy

and given our blessing. By means of such rods we have, for example, found that mathematics falls short of realizing the social needs through non-specialized or general education. We have used these artificial rods to conclude that physics and chemistry are desirable in the training of the individual for industries but not for citizenship. We have even gone further and endowed certain subjects with all the desirable attributes necessary for the attainment of social needs of the individual, and condemned other subjects as lacking of these same attributes.

Such a procedure clearly neglects the man for the subject. For, how did mathematics, physics or chemistry happen to be an integral part of the curriculum? Was it because each of these subjects was evaluated and measured in the past, as we attempt to do at the present, and found worthy of a place in the educational system, or was it because each of these subjects grew gradually as a result of man's effort to achieve certain objectives in his struggles for the betterment of himself and of the social group? And if each of these subjects once had such a worthy mission in education, what has transpired that they are no longer capable of performing that mission?

The proper approach to the analysis of the problem seems to be a recognition of the man's relations to the civilization he has created, and an evaluation of the contributions which the natural sciences have made to the maintenance and the advancement of that civilization. We may as individuals, educated or not, either take our civilization for granted and consider it as a gift of nature, as we do the air we breathe, or we may become aware of the fact that civilization is man-made and is a product of human labor. If the former attitude is accepted, and we sadly reflect that it is the attitude of many of us, we have neither an appreciation of its genesis nor a comprehension of its laws of growth and maintenance. If however, we are fortunate to have established the latter attitude then such an endeavor produces a realization of the mighty obligations which the benefits of our civilization impose upon us to preserve it as a great inheritance. To meet these obligations we must have an adequate understanding of the role which natural sciences and mathematics have played in the development of civilization.

If we attempt to deal with man in his relations to civilization we are forced to acknowledge the fact that man is a supreme reality. To understand him we must understand his activities, the activities which are motivated by these relations to civiliza-

tion. His activities raise certain problems which involve principles of ethics, of social institutions, of economics, of philosophy and religion, of politics, and of industry and science. To meet the social needs of the individual, education must in some measure find answers to all of these problems. Solution of isolated problems in special fields of endeavor may or may not contribute to an understanding of man's relations to civilization. On the other hand, an analysis of these relations in terms of all the subjects which constitute the curriculum is apt to result in a coordination of the educational efforts of the various subjects toward a satisfactory general solution.

In its relations to man's activities, education so far has had a twofold function. This is manifested in the distinctions between specialization and general education. Specialization is primarily industrial education in purpose and method. General education can be humanistic both in purpose and method, but need not be that. When general education is not primarily humanistic it ceases to be general. The distinction between these two types of education needs to be pointed out. They are real and distinct, and do not depend upon the particular name we use in describing an educational system.

Industrial education, that is, education for specialization, endeavors to equip the individual for the functions of life. It detects as early as possible the presence of special natural gifts or propensities which tend to indicate or predict the specific vocation for which the individual is to become best qualified. The general education, that is, humanistic education, on the other hand, aims to equip the members of the social order, all of them in so far as possible, with the qualities and characteristics that the social order demands, qualities and characteristics that are essential to the maintenance of the social order and to its betterment. The equipment provided by humanistic education takes the form of development of the common human powers and capacities, of accentuation of the desirable qualities and of discouragement and elimination of the undesirable qualities; desirability or undesirability of a quality being judged from the view point of the welfare of the social group. Contacts and experiences are provided by education for the development of the desirable qualities in terms of actual learning situations.

The learning situations of the industrial education are concerned primarily with individualities. Industrial education presents a picture in which the world is an immense camp of

industries, each industry calling for special kind of training and specialization.

The learning situations of humanistic education aim differently. They aim at the attainment of some standard of excellence in those capacities which constitute our common humanity. As human beings we have certain common instincts, traits, powers, and capacities, some of which we hold in common with sub-human animals, others which are intrinsically human in nature. Humanistic education endeavors to develop the distinctly human attributes of the individual.

The curriculum of mathematics can be humanized if, and only if, it can be shown that mathematics can contribute in a definite manner toward the development of our common human attributes.

Without attempting to give a precise definition, even if that were possible, of the common attributes of man, we can identify them, somewhat vaguely to be true, but identify them, nevertheless, and classify them. In all his stages of development man has exercised the following faculties, which we may term as constituting our common humanity. Man has a literary faculty, that is, a sense for language and expression; an historical faculty that is, a sense for the past or for the value of experience; a scientific faculty, that is, a sense for prediction or for understanding the natural laws; a mathematical faculty, that is, a sense for logical and rigorous thinking in dealing with quantities and ideas; a political faculty, that is, a sense for fellowship, for cooperation, for justice; a religious faculty, that is, a sense for the mystery of divinity and the future of life; a philosophical faculty, that is, a sense for generalized knowledge and universal truth; and an esthetic faculty, that is, a sense for beauty, for harmony, and for order. Education, whatever its purpose or scope, should concern itself with the development of these our common faculties and with the attainment of some degree of excellence in their applications, and with the acquisition of a vivid and abiding sense of their authority in the conduct of life of the individual in his relationships to the social order.

The two types of education may supplement each other, but can not substitute for each other. If the natural sciences are to train only specialists they may succeed in producing craftsmen, but not educated men, trained specialists, but ignorant citizens, expert scientists, but not socialized human beings. It is true that in each subject the individual receives a training in some par-

ticular capacity, but the question of educational importance is, is that the only capacity which that subject can train? Must mathematics be concerned only with the development of the mathematical faculty of man, or is it possible for it to make contributions toward the development of scientific faculty, the historical faculty, the esthetic faculty, and so on? What are the educational contributions of the physical sciences to the development of other than the scientific faculty of man? Of the many common human faculties, it is quite reasonable to assume, that the natural sciences and mathematics, under favorable learning situations, can contribute a great deal to the development of some of them, and a lesser degree to the development of others. But those contributions must be definite, must be attainable in actual learning situations as actual learning products. The following report may be of interest in showing whether or not the traditional courses in sciences and mathematics are equipped to carry out the functions of humanistic education. The report was prepared in 1939 by the Committee on the Improvement of Science in General Education, appointed by the American Association for the Advancement of Science. The data are taken from a questionnaire sent to colleges, universities, and teacher training institutions.

"1. Do you consider the conventional introductory courses in physics, chemistry, and mathematics as represented by a majority of current textbooks and laboratory manuals: (a) are in general satisfactory for the non-specializing student?"

The replies are:

physics: yes—64, no—106, uncertain—26;
chemistry: yes—65, no—122, uncertain—22;
mathematics: yes—51, no—130, uncertain—24.

These numbers are significant enough in all three subjects to show that less than one-third of the teachers of these subjects find the traditional courses satisfactory for the non-specializing student. The other two-thirds, a majority, feel that they are not satisfactory, and a few are uncertain.

"(b) could be significantly improved for the non-specializing student?"

The replies are:

physics: yes—119, no—33, uncertain—39;
chemistry: yes—137, no—25, uncertain—38;
mathematics: yes—136, no—31, uncertain—32.

Again about two-thirds of the teachers in each subject feel that the courses could be significantly improved.

These two questions alone indicate that a large majority of the teachers of physical sciences and mathematics feel the curriculum of the sciences and mathematics should be humanized and that the present traditional courses are inadequate for that humanization. The rest of the report is concerned with a determination of the manner in which a reorganization of these courses may be brought about, and the specific concepts and processes which lend themselves to a program of general education. For example: "What do you believe are the most significant contributions which a study of physics, chemistry, or mathematics should make for those students who are not to specialize in them? (1) Develop the ability to do critical thinking":

physics: yes—165, no—30;
chemistry: yes—176, no—20;
mathematics: yes—168, no—27.

Critical thinking is one of the common attributes of man that no particular field of study can claim as its own.

The report is also concerned with means of bringing about this organization and with methods of instruction to attain the general educational aims of the sciences. For example: "What are some of the things which you would like to have developed in connection with these problems?" The development of methods for discovering the particular needs and interests of students and for selecting science content and teaching procedures to meet those needs and interests!

physics: yes—122, no—51;
chemistry: yes—123, no—54;
mathematics: yes—103, no—61.

We might go on and present other evidence from that report which constitutes the opinions of the teachers of science and mathematics concerning the problem of humanizing the curriculum in the sciences. However, we have presented sufficient information to show the trends followed by these opinions. First, the majority feel that the science curriculum should be humanized in terms of such concepts as are applicable to the non-specializing student. Second, the present courses and texts are inadequate for that new and exclusive educational program.

Third, methods and means should be devised for the solution of the problem both in the solution of instructional materials and in methods of instruction and evaluation.

But you ask can it be done? Is there a satisfactory solution to the problem? Can the curriculum of mathematics be humanized? Can the curricula of the natural sciences be so organized and so presented and developed that they will make notable contributions to the attainment of the general educational objectives? We are in a position to state first that no problem is ever solved by only wishing for a solution. Some one must actually attempt a solution. If he fails he has at least called attention to the significance of the problem. Others may succeed where he failed. At the present time there are many institutions where serious efforts are made to humanize the curricula of natural sciences, among them the institution which I represent. This important problem cannot be solved over night, subjectively. It takes much time and much experimentation. We at Wright have a feeling that we are using the right methods of attack. The signs that we can perceive all tend to show that a solution can be worked out so that the natural sciences and mathematics can and will fulfill their unique functions in general education, in the attainment of those educational objectives which strive to develop the common attributes of man in the realization of those trainings which give the man, the educated man, a full sense of responsibility of his relationships to society and civilization.

POOR NUTRITION BLAMED FOR ARTERY HARDENING

Hardening of the arteries is due fundamentally and directly to undernourishment of the walls of the arteries, in the opinion of Dr. W. C. Hueper, New York City. Support for his theory appeared in experiments in which he treated rats with the chemical, erythrol tetranitrate. This substance lowers both blood pressure and the amount of oxygen in the blood. As a result, nourishment of the arteries might be impaired. In the experiments, Dr. Hueper reported, the rats after treatment with this substance developed sclerotic or hardened arteries of the heart, lung, kidneys and certain other organs.

SCIENCE PHOTOGRAPHS TINY MOLECULES

By using beams of fast electrons scientists are now able to photograph tiny molecules, Dr. Peter J. W. Debye, Dutch Nobel Prize winner in chemistry, said at his lecture at Cornell University. Molecules are so small that a tiny drop of benzene contains several billions of them.

THE PLIGHT OF HIGH SCHOOL PHYSICS*

IV. The Languishing Laboratory

H. EMMETT BROWN

Teacher of Science, Lincoln School, Teachers College, and Instructor in Teaching of Natural Sciences, Teachers College, Columbia University

Recent years have seen an increasing attack upon the use of the laboratory in high school physics. We do not need to go into the bases for this attack here—economy programs, the findings of certain experimental investigations into the relative merits of individual laboratory and demonstrations methods, etc. Suffice to say, physics has been peculiarly open to such attacks. Expensive equipment has been cited as a cause—and probably with some justice. But is that the sole reason? Let us examine our practice a bit.

Jim Smith comes into the physics laboratory of A——— High School. He was told during the previous day's class period to read over Experiment 14, "The Composition of Forces," in Blank, Blank and Blank's *Laboratory Manual in Glorified Physics*. Now this experiment is worthy of the veneration that is accorded the aged in all well-regulated societies. Under this same title, and in only slightly different form it appeared in the *Harvard Descriptive List of Experiments* in 1886, and in several other of the early standardizing reports. The same is true of many of the other experiments in the manual. But Jim is unaccountably lacking in his reverence for fusty senility. He has not read the experiment! Educators are aware that the laboratory trains in the scientific method of solving problems—that it, the laboratory, is a place in which "to put questions to nature." But Jim, not being an educator, doesn't know this. And so he proceeds to plod stodgily through the experiment, taking his directions from the manual line for line, in cook book style. He has no problem to solve—except the personal one of getting through the experiment with a minimum of labor and thought. But he *does* have a little bad luck. He draws the angles of his force parallelogram carelessly and the resultant does not equal the equilibrant. Jim doesn't notice the error. Why should he? But his teacher, Dr. Pangloss, does. "*You will have to do the experiment over.*" But the laboratory period is nearly over.

* Previous articles in this series have appeared in the issues of this magazine for June, 1939 (p. 58); Dec., 1939 (p. 840); Feb., 1940 (p. 156); and April, 1940 (p. 368).

After-school hours will be required for the second performance. Here at last is a real problem—how to avoid the extra hours! Jim is by no means stupid. A little juggling of figures, a little consultation with his classmates, and Jim is able to submit a drawing and force values which elicit Dr. Pangloss's approval. A written report is required. This is turned in the next day. In it Jim concludes that he has "proved" that the resultant always equals the equilibrant.

We should not conclude that this experience has been without value for Jim. True it has rather stifled whatever interest in experimentation he has retained through previous laboratory periods. He has received no insight into the scientific method. But he *has* had something to *do* during the period. And that last little brush with the teacher *was* interesting. For, not knowing that the laboratory teaches "mental rectitude," Jim has been smart enough to "doctor" his results so that the values for resultant and equilibrant are neither so close as to excite suspicion, nor so much in variance as to evoke another demand for repetition.

We do not need to raise the question of whether Jim's laboratory experience, characterized by stereotyped experiments, cook book procedure and inculcation of undesirable habits and attitudes, typifies the average physics laboratory or no. Let us instead turn our attention to the far more important question of what may be done to improve our laboratory practice.

It is beyond the scope of the present article to enter into any discussion of the relative merits of the individual laboratory and the demonstration method. Suffice to say, considered opinion now seems to favor the use of both methods. Both are needed. Neither suffices alone.

If the physics laboratory is to realize on its potentialities what is needed most fundamentally is a new point-of-view. We need to envisage our laboratories not only as an educational workshop where understandings and abilities and attitudes may be developed but also as a therapeutic studio where the more personal needs of individuals may be met. Let us first consider the latter of these functions briefly.

Here is an individual whose pressing need, at the moment, is not for knowledge or skill, but for the assurance that he *belongs*, that he is a vital part of the group. The wise science teacher, sensing this need, will make sure that that child becomes part of a group working together on some common task in the accom-

plishment of which he may make a contribution that is recognized of value by the other members of the group. Or again, it becomes apparent that what another member of the class needs is to learn to direct his own activities more satisfyingly and efficiently. The teacher again tries to engage the student's interest in some laboratory activity for which the planning of his own time is an essential step.[1]

It will be noted that in the instances cited, the exact character of the laboratory experience was not indicated. Indeed it is probably true that laboratory experiences with almost any unit of science work may be used to meet these more personal needs. Let no one take this lack of exact definition as an indication that such laboratory experiences are of slight moment, however.

Perhaps the most important change which we must effect if our physics laboratory is more effectively to develop desirable understandings, attitudes and abilities is to use this, our educational workshop, as a place for solving problems. By problems I mean not those set by the teacher or the laboratory manual, but problems which the pupils themselves are individually interested in solving. Now this is not easy. It requires a degree of individualization of laboratory work that is not common. It also requires that pupils become sensitized to the many problems of their daily lives which they can "tackle" in the laboratory, and truly creative in their setting up of methods by which the laboratory attack may be made. Such pupil activity can only come as the result of practice in the use of this method. We may begin by asking our class to solve some relatively simple problem in the laboratory. "How do we determine the direction of sounds when you cannot see the source?" "What is the frequency of the note produced by an unmarked tuning fork?" The objection may be raised, and rightly, that these are not pupil selected problems. Our justification is in terms of the practice needed. The next step may be to place before the student, perhaps in mimeographed form, a short excerpt from a text or other book. Each student is asked to list all the problems that are suggested by the material. He is then asked to suggest methods by which he thinks one or more of the problems may be solved. Then he proceeds along one or another of the

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[1] This aspect of the laboratory has been developed at greater length in the following:

Thayer, V. T. and Others. *Science in General Education*. D. Appleton-Century, New York, 1938, pp. 52, 133–134, 246.

Slavson, S. R., "Values of Laboratory Experience. "*Childhood Education* 15: 341–346; 1939 (April).

lines he has suggested. A next step may be a repetition of this last one, but with problems which he brings in without the stimulus of the printed excerpt. All, or a portion of the entire routine may need to be repeated but eventually, if the teacher encourages such work and affords the necessary time, there should commence to be an increasing number of students who will try their wings at the new venture. On some one or two occasions during the year, when a majority of the class have problems of a more difficult nature that they would like to work upon, it may be necessary to allow laboratory work of this nature to go on for a number of consecutive class periods.

At this point it may be desirable to forestall certain objections that may be raised. It may be said that students will not behave as described, that they will not sense problems for laboratory solution, nor carry out the necessary laboratory procedures. *But they will.* The above is no theoretical discussion. It has been done in classes of a number of different teachers.[2] Or again, what of the laboratory time—is there not danger of the periods being wasted because of the student's stumbling starts? But let us not forget Jim Smith. The student's time is being wasted in most of our conventional laboratory work, anyway! We have little, if anything to lose, and the potentiality of much good to gain by beginning this type of laboratory work as fast as the size of our classes and other conditions will permit. Perhaps we may begin with only a few students. But let's begin.

Once having wedged in the kind of experimentation in which the answer to the problem is not known in advance, the door should be open to other kinds of laboratory experience as well. Bearing in mind our dual concept of the function of the laboratory, we may wish to allow students to make pieces of apparatus, or to set up exhibits of a scientific nature. A pile-driver, various pieces of electrical apparatus, a wave-motion apparatus constructed from cobalt-chrome magnets, models of gear systems, or the lever system of the human arm and various display cases, e.g., one on stage lighting, the lights of which went through a cycle of operation after one press of a button outside the case, are among those which have, at various times, been constructed by students in my classes. We may wish, of course, to require a report dealing with the principles on which the device operates, but that will vary with the individual case.

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[2] Unfortunately few articles have appeared which describe such work. That fact constitutes the justification for the single reference given below.

Brown, H. Emmett, "A Project in Physics." *Science Education* 18: 42–45; 1934 (Feb.).

In certain cities, opportunities are given young people to exhibit in a sort of science fair displays or apparatus they have built. The preparation of material for such fairs should not be thought of as an exclusively extra-curricular activity but may well occupy a part of the time devoted to laboratory work.

The practice of a fixed scheduling of laboratory periods is probably the basis for an equally rigid attitude toward the use of those periods. It should be possible at any time to introduce short, individual, experiments, informal in character, and often not dignified by a special write-up, which are used to clear up some point that has risen in connection with class discussion, or which have been foreseen as valuable class experience. In this category may be placed such experiments as that designed to show that the image on the retina of the eye is inverted (hold a pin vertically and close to the eye; gaze past the pin, through a pinhole in a card toward a bright light), simple experiments with refraction, experiences with the use of lenses to form images, etc. Such experiments, usually qualitative and descriptive in character, may be used to provide the background of experience out of which develop science understandings. They are performed with a definite purpose, and at a time when they are demanded by the character of the work that is being done by the class, not by the schedule of laboratory periods.

Two other kinds of experiments are suggested by previous articles in this series. In "Peccant Psychology" (SCHOOL SCIENCE AND MATHEMATICS 40: 156–160, Feb. 1940) the necessity for selecting material close to the student's immediate interests was stressed. In the laboratory, therefore, let us include a much larger number of experiments designed to relate the physics phenomena to the student himself, e.g., experiments on the effects of noise on the nervous system, on the psychology of color, on the power rating of the human body, on the body as a machine, etc. In one such study a student was attempting to find what emotional associations certain colors have. She had mimeographed a rather long list of words and asked the class to associate each word with one of the colors—red, blue, yellow, green. A rather surprising unanimity of rating was discovered. In another experiment various colors were projected on a screen while such orchestral compositions as "To a Wild Rose" and Chopin's "Funeral March" were played. The point was to try to determine what color is associated with a certain mood evoked by music.

In "Water-Tight Compartments" (SCHOOL SCIENCE AND MATHEMATICS 39: 840–845; Dec. 1939) the desirability of breaking down the rigid compartmentalization of the subject through the use of the energy theme as an organizing principle was suggested. We therefore include a number of experiments which are designed to bring out the interconvertibility of energy forms. Among the best of such experiments are those in which we measure the over-all efficiency of certain devices—an electric motor, a steam engine heated by a burner using gas at a constant rate, a model generating plant, a gasoline engine, etc. Perhaps best of all would be to measure the efficiency of the human body considered as a heat engine performing a task. Such an experiment is beyond most high schools because of the elaborate equipment required. (Calculations made from data obtainable in texts show that the human body has an efficiency of about 40% when working at its optimum rate.)

If we are to make these and other desired changes in the laboratory work in physics we must not hesitate ruthlessly to extirpate many experiments in our present list. All verifications, proofs of laws (as though it ever were possible to "prove" a law in science from a few sets of hurriedly assembled measurements), detailed and overly precise quantitative experiments and experiments dealing with apparatus which has neither historical interest nor modern usage, are certainly in this category as far as individual experimentation is concerned. Neither must we hesitate to try new ways of using the laboratory, either for fear of failure (remember the meager value of traditional laboratory work) or because of the fancied demands of the College Entrance Examination Board or other standardizing body. Success is only waiting our bold venture!

CHEMICAL FORMULATOR

This is a gadget designed to assist the beginning student of chemistry in writing correct formulas, and at the same time to impart an understanding of valence. It consists of two concentric disks of cardboard of different diameters which rotate relative to each other around the common center. By the simple procedure of matching valence bonds (represented by red lines on the inner and outer disks), more than 500 formulas can be found; also the common names of all hydrogen compounds are automatically signalled when the valence bonds are matched.

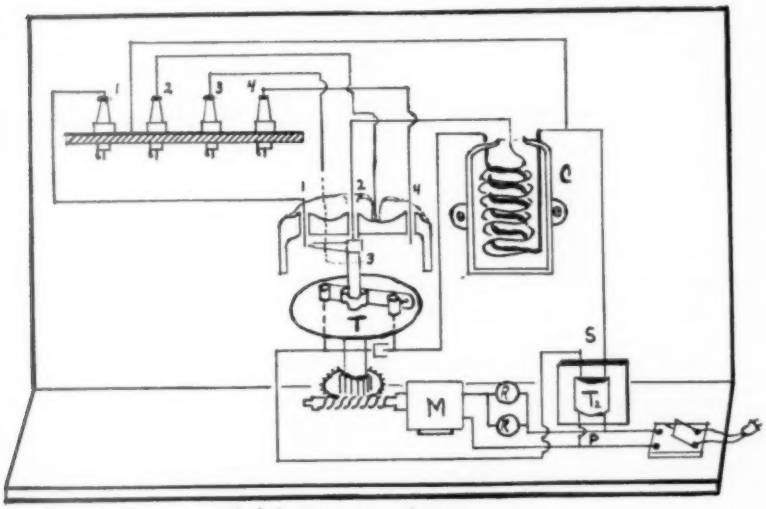
This new instructional tool may be obtained from Cambosco Scientific Company, 37 Antwerp Street, Brighton Station, Boston, Mass.

AUTOMOBILE IGNITION ASSEMBLY

F. W. Moody
Cleveland High School, St. Louis, Missouri

After hearing a description of the induction coil and timer and an explanation of the way they function in the ignition system of an automobile, classes like to see some demonstration of the system in operation.

An assembly designed to show this and ready for instant use, is here shown. The timer "T," coil "C," spark plugs and con-



Model Ignition System.

necting wires, were a contribution from an automobile dealer and were taken from a wrecked car.

A small discarded low voltage motor "M" was fixed up as a driver to rotate the timer. A large wood screw whose head was sawed off and whose threads were filed deeper was coupled to the motor shaft and served as a worm to drive the spur gear which was attached to the timer shaft. The spur gear may be obtained from an old alarm clock.

To make the assembly self contained so that a battery is unnecessary an old step down radio transformer "T_2" partly rewound, is used to supply the current for the ignition coil.

Two lamps in parallel are connected in series with the motor to act as a resistance and allow the motor to be used on the 110

volt circuit. The whole assembly may then be run from the regular power supply.

The plugs are connected to the timer in such a way that they flash as in a four cylinder engine, in the order, 1, 2, 4, 3.

To operate the device all that is necessary is to connect the cord to the power circuit and close the switch. The flashes are sufficiently bright and the intervals long enough for the class to be able to follow. By means of the assembly it takes only about two minutes to demonstrate the principles involved in the ignition system of the automobile. The saving in time and effort is apparent as few teachers could take the time to set it up for one or two classes once a term.

THE PSEUDO-ISOSCELES TRIANGLE

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By an interesting use of a theorem of Stewart, J. J. Corliss[1] derived the equation of condition that a triangle be pseudo-isosceles which we write as

$$f(c)=c^3-c^2(a+b)+3abc-ab(a+b)=0. \qquad \text{(A)}$$

Designating the sides of triangle ABC by a, b, c, the perimeter and area by $2s$ and $\triangle$, the external bisectors by AD' and BE', the circumcircle by $O(R)$ and the tritangent circles by $I(r)$, $I_a(r_a)$, $I_b(r_b)$, $I_c(r_c)$, we present some properties of this scalene triangle with the equal angle bisectors, whose existence was first noted by Steiner.[2]

When $AD'=BE'$ and D' and E' are on BC and CA prolonged, then triangle ABC is scalene. The angles ACB and ACD', opposite the equal lines AD' and BE' are supplementary but not equal; for if $\angle C=90°$, the sum of the angles of triangle BAE' would be greater than 180°. Also CI_c is an external bisector in triangle ACD' and an internal bisector in triangle BCE' and triangles ACD' and BCE' are not congruent.

If $AB=AD'=BE'$, then, since $\angle D'=\angle B$, we have $\angle C=3\angle B$ and $\angle A=11\angle B$. Hence $\angle B=12°$, $\angle C=36°$ and $\angle A=132°$, the simplest known example of this triangle.

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[1] SCHOOL SCIENCE AND MATHEMATICS, November 1939.

[2] For the history, bibliography and proofs of the first case of the Lehmus-Steiner Theorem, see SCHOOL SCIENCE AND MATHEMATICS, June 1939.

Properties:

1. Since $CE':AE'=a:c$ and $BD':CD'=c:b$, $a>c>b$.

2. The positive real root of (A) lies between $a-b$ and a, for $f(b)=ab(b-a)<0$, $f(a-b)=-2b^3<0$, $f(a)=ab(a-b)>0$.

3. Equation (A) may be written $f(c)=c(c-a)(c-b)+ab(c-a)+ab(c-b)=0$ which divided by $ab(c-a)(c-b)$ gives

$$\frac{c}{ab}+\frac{1}{c-b}-\frac{1}{a-c}=0, \quad \frac{1}{c-b}<\frac{1}{a-c} \text{ and } c>\tfrac{1}{2}(a+b).$$

4. Now $(c-a)(c-b)<0$ or $c(a+b)>(c^2+ab)$. Writing equation (A) in the form $c(c^2+3ab)=(a+b)(c^2+ab)$ and dividing, we have $c^2<a^2+b^2-ab$ and $\angle C<60°$.

5. The relation,

$$\frac{s-a}{a}\cdot\frac{s-b}{b}=\frac{(s-c)^2}{c^2}, \tag{B}$$

given by Corliss is equivalent to

$$\sin^2\tfrac{1}{2}C=\sin\tfrac{1}{2}A\,\sin\tfrac{1}{2}B, \tag{C}$$

since

$$\left[\frac{(s-a)(s-b)}{ab}\right]^2=\frac{(s-b)(s-c)}{bc}\cdot\frac{(s-a)(s-c)}{ac}.$$

6. The quadrilateral $AIBI_c$ is cyclic, since its opposite angles are supplementary. By drawing $IL\perp CA$ at L, $I_cL'\perp CA$ prolonged at L', cutting circle AIB at M, we have $CL=s-c$, $LL'=c$, $IM\parallel LL'$ and $I_cM=r_c-r$.

From the similar triangles CIL and II_cM,

$$(s-c):c=r:(r_c-r)=CI:II_c.$$

In like manner,

$$(s-b):b=r:(r_b-r)=BI:II_b.$$

$$(s-a):a=r:(r_a-r)=AI:II_a.$$

Since I_aA, I_bB, I_cC, are altitudes of triangle $I_aI_bI_c$, $AI\cdot II_a=BI\cdot II_b=CI\cdot II_c$, whence $CI^2\cdot II_c^2=BI\cdot II_b\cdot AI\cdot II_a$. Equation (B) then takes the form

$$(r_c-r)^2=(r_a-r)(r_b-r). \tag{D}$$

Also

$$\frac{CI^2}{(II_c)^2}\cdot CI^2\cdot II_c^2=\frac{BI}{II_b}\cdot\frac{AI}{II_a}\cdot BI\cdot II_b\cdot AI\cdot II_a,$$

whence

$$CI^2 = BI \cdot AI. \tag{E}$$

Likewise, $II_c^2 = II_b \cdot II_a$.

7. In the circumcircle O let d_a, d_b, d_c, be the chords of half the arcs subtended by the sides a, b, c. By drawing the diameter $POQ \perp AB$, cutting AB at M', where P is the midpoint of arc AB and Q the midpoint of arc ACB, and AP and AQ, we have from the similar triangles CIL and APQ that $CI \cdot d_c = 2Rr$. In like manner, $BI \cdot d_b = 2Rr = AI \cdot d_a$. Now $CI^2 \cdot d_c^2 = 4R^2r^2 = BI \cdot d_b \cdot AI \cdot d_a$. Hence from (E) $d_c^2 = d_a \cdot d_b$.

8. Triangles AIL and BII_c are similar, since $\angle IAB = \angle BI_cI$ and $\angle ALI = 90° = \angle I_cBI$. Hence $AI : II_c = r : BI$ or $r \cdot II_c = AI \cdot BI$. But $CI : II_c = r : (r_c - r)$ or $(r_c - r) \cdot CI = r \cdot II_c$. As $CI^2 = AI \cdot BI$, then $CI = r_c - r$.

9. Since $CI : II_c = r : (r_c - r)$, then $(r_c - r) : II_c = r : (r_c - r)$ and

$$II_c = \frac{(r_c - r)^2}{r}.$$

In like manner,

$$CI_c = \frac{r_c(r_c - r)}{r}.$$

10. By means of equation (D) and $abc = 4R\triangle$, we have

$$\left(\frac{r}{r_c - r}\right)^3 = \frac{r^2}{(r_c - r)^2} \cdot \frac{r}{(r_c - r)} = \frac{s(s-a)(s-b)(s-c)}{abcs}$$

$$= \frac{\triangle^2}{4R\triangle \cdot \triangle / r} = \frac{r}{4R}.$$

Then $4Rr^2 = (r_c - r)^3 = CI^3$.

11. Equation (A) may be written $(c^2 + ab)(s - c) = abc$. Then

$$c^2 + ab = \frac{abc}{s-c} = 4R\triangle : \frac{\triangle}{r_c} = 4Rr_c.$$

12. From $r_c(s-c) = \triangle = \frac{1}{2}ch_c = rs$ and $s(s-a)(s-b)(s-c) = \triangle^2 = rs \cdot r_c(s-c)$, we obtain

$$s - c = \frac{ch_c}{2r_c}$$

and $rr_c = (s-a)(s-b)$.

Using equation (B),

$$\left(\frac{ch_c}{2cr_c}\right)^2=\frac{rr_c}{ab}=\frac{rr_c}{2Rh_c}$$

and $R:r=2r_c^3:h_c^3$.

13. Since $\angle ACI_c=\angle BCI$ and $\angle CBI=\angle ABI=\angle AI_cC$, then triangles BIC and AI_cC are similar and $CI\cdot CI_c=ab$.

The quadrilaterals $CIAI_b$ and $CIBI_a$ are cyclic and $CI_a\cdot I_aI_b=AI_a\cdot II_a=BI_a\cdot II_c$, $CI_a(CI_b+CI_a)=II_a(AI+II_a)$, and $CI_a\cdot CI_b=CI\cdot CI_c$.

The triangles CAI_c and CBI_c give:

$$AI_c:\sin\tfrac{1}{2}C=b:\sin CI_cA=b:\sin\tfrac{1}{2}B,$$

$$BI_c:\sin\tfrac{1}{2}C=a:\sin BI_cC=a:\sin\tfrac{1}{2}A,$$

$$AI_c\cdot BI_c:\sin^2\tfrac{1}{2}C=ab:\sin\tfrac{1}{2}A\sin\tfrac{1}{2}B.$$

Hence by equation (C),

$$AI_c\cdot BI_c=ab \quad \text{and} \quad CI\cdot CI_c=CI_a\cdot CI_b=AI_c\cdot BI_c=ab.$$

14. From triangles AI_bB and AI_aB,

$$AI_b:\sin\tfrac{1}{2}B=c:\sin AI_bB=c:\sin\tfrac{1}{2}C$$

$$BI_a:\sin\tfrac{1}{2}A=c:\sin AI_aB=c:\sin\tfrac{1}{2}C$$

$$AI_b\cdot BI_a:\sin\tfrac{1}{2}A\sin\tfrac{1}{2}B=c^2:\sin^2\tfrac{1}{2}C.$$

Hence by equation (C),

$$AI_b\cdot BI_a=c^2.$$

15. Since

$$AI=\frac{r}{\sin\frac{1}{2}A},\quad BI=\frac{r}{\sin\frac{1}{2}B},\quad \angle AIB=90^\circ+\tfrac{1}{2}\angle C$$

we have

$$cr=2\triangle AIB=AI\cdot BI\cdot\sin AIB=\frac{r^2\sin AIB}{\sin\frac{1}{2}A\sin\frac{1}{2}B}$$

$$r=\frac{c\sin\frac{1}{2}A\sin\frac{1}{2}B}{\sin AIB}=\frac{c\sin^2\frac{1}{2}C}{\sin AIB}=\frac{c\cos^2 AIB}{\sin AIB}.$$

16. Since the projection of $II_c=LL'=c$, and $c=2R\sin C$,

$$II_c = \frac{IM}{\cos \frac{1}{2}C} = \frac{c}{\cos \frac{1}{2}C} = \frac{2R \sin C}{\cos \frac{1}{2}C} = 4R \sin \tfrac{1}{2}C.$$

17. Finally, $\angle PAI = \frac{1}{2}(\angle A + \angle C) = \angle AIP$, $PI = PA = PB$, $\angle PBI_c = \frac{1}{2}\angle A = \angle PI_cB$, and $PB = PI_c$. Hence, for any triangle, P, the midpoint of arc AB is the midpoint of II_c.

Draw $I_cN \perp AB$, meeting the parallel to AB through I at N on circle AIB. Then $PM' + r = \frac{1}{2}I_cN = \frac{1}{2}(r_c + r)$ and $PM' = \frac{1}{2}CI$.

Also

$$\frac{PM'}{PA} = \sin PAB = \sin \tfrac{1}{2}C = \sin AQP = \frac{PA}{2R}.$$

Hence

$$PM' = 2R \sin^2 C \qquad \text{and} \qquad CI = 4R \sin^2 \tfrac{1}{2}C.$$

CRITERIA FOR SELECTING CHEMISTRY EXPERIMENTS

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A major problem that confronts high school chemistry teachers as well as instructors in the other sciences is the periodic necessity of selecting proper laboratory manuals, or the proper equivalent, for their chemistry classes. The degree of success obtained in the laboratory is determined to a great extent by the quality of the experiments as set up in such books. Uninteresting and faulty laboratory work can often be traced to poorly chosen experiments. This does not necessarily reflect upon the experiment but rather may be caused by the failure of the teacher to scrutinize prospective material. Manuals are too frequently selected on the basis of whether or not they contain experiments dealing with certain topics with disregard to the quality of the exercises. It is well worth the extra work in ascertaining definitely the usefulness of a book by carefully comparing each experiment with certain pre-determined standards. Final selection should be made on the basis of this rating. Quality should never be sacrificed for the sake of several experiments which may be wanted for a course. It is much better to supplement a manual with one or two duplicated exercises.

The reasons for a careful check of laboratory manuals are

numerous. It is an essential routine duty because of the large number of books available, the variation in the presentation of the problems, the many ways in which a principle may be illustrated, the failure of some writers to see the true purpose of laboratory work, clearly defined guiding principles are the best bases of proper judgment, and finally, with the modification of subject matter to meet student interests and needs there is a need of selecting and setting up new experiments.

The following criteria are offered as a means of rating experiments or, in positive forms, as guiding principles in setting up new experiments.

1. Does the experiment illustrate an important chemical principle?
2. Is the principle important enough relative to the objectives of the course to warrant an experiment?
3. Does the principle involved justify a separate experiment or could it be included as a minor part of another?
4. Is the complexity of the principle such that it can be presented to the largest number of students by the experimental method?
5. Does the experiment include clear and accurate directions, and steps that will lead to the proper conclusions?
6. Can the conclusion be reached without facts that must be obtained from sources other than the results? (This does not refer to facts and skills set up as requirements for the work.)
7. Are the related facts or exercises to be supplied so placed as not to affect the conclusion?
8. Are the skills to be developed useful in relation to the objectives of the course?
9. Does the experiment supply training in skills and supply facts that are included in the objectives of the course?
10. Is the experiment so designed as to make it necessary for the students to think and work their way through the experiment?
11. Does the experiment make the students state their own conclusions in their own words?
12. Does the experiment represent the quickest and easiest way of arriving at the conclusion at that particular subject level?

When you change address be sure to notify Business Manager W. F. Roecker, 3319 N. 14th Street, Milwaukee, Wis.

AN IMPROVED GAS LAWS APPARATUS*

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Many teachers who have been dissatisfied with the simple Boyle's Law apparatus and who did not wish to use the more elaborate type, will welcome the advent of an inexpensive yet improved gas laws apparatus. The first cost of the device is

Boyle's Law—Fig. 1

Charles' Law—Fig. 2

most moderate and the mercury requirement is small. In the trial runs checked by the writer only 23 cc. was used. With ordinary care none should be lost and since it need not be removed both the mercury and the apparatus should remain clean indefinitely.

The novel arrangement of the apparatus is apparent in the

* The Gas Laws Apparatus here described was designed by the Central Scientific Company, 1700 Irving Park Blvd., Chicago, Ill., and retails at $8.50.

illustrations. Briefly it consists of a J-shaped manometer tube with a connection at the bottom of the J to a large tube or well.

In studying Boyle's Law (Fig. 1) the mercury is poured into the well and the apparatus inclined and shaken to bring the mercury in the open and closed arms to the same level. The enclosed volume of air is then at atmospheric pressure. To increase the pressure on this air, the wooden rod is pushed down into the mercury in the well. Since the rod is only slightly smaller than the inside diameter of the well, the mercury rises rapidly in it and obviously in the small open tube connected to it. This of course compresses the air in the closed tube. A series of readings is taken to show the effect of a change in pressure on the volume of a given amount of gas. These readings are obtained by pushing the rod deeper into the mercury. Because the apparatus is a modification of the usual form the data may be recorded in the data forms found in most of the regular laboratory manuals, with very little change.

The most interesting feature of the device is its ability to illustrate "Charles' Law." In this demonstration (Fig. 2) the whole apparatus is placed in the tall jar furnished for the purpose. Cold water is poured into the jar until it rises above the top of the closed tube. The stick is pushed down until the difference in the two mercury levels is about 20 cm. This difference or pressure must be maintained throughout the experiment. The volume of the air is measured and the temperature which is obviously the same as that of the water, is also recorded. The cold water is then siphoned out and warm water poured in. After waiting for a few minutes for the temperature to become constant, the new volume and temperature are taken and recorded. This procedure is repeated several times using increasingly higher temperatures each time.

The data obtained may be treated in several ways. It may be plotted on graph paper to show that the volume of a gas at constant pressure is directly proportional to the absolute temperature; or the original volume and the two temperatures may be used to compute the new volume which is then compared with the observed volume.

It is also possible to hold the volume constant and show the effect of a change in the temperature of a gas, on the pressure exerted by that gas.

This excellent piece of apparatus will be a valuable addition to the equipment of any laboratory.

TEACHING MATHEMATICAL INDUCTION

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The purpose of this article is to describe a method of teaching Mathematical Induction which includes some new features, original with the author and developed with the cooperation of his students. The method consists essentially of three parts. First, the students become familiar with this type of reasoning by using it to prove simple well known formulas. Second, they acquire an understanding of the full meaning of each step. Third, by the use of a short method, they are able to prove the formulas more easily than by other methods.

Let us see how a well known formula may be proved by Mathematical Induction. The cost formula, $c=5n$, is the formula for the cost of n articles when the price is five cents each. The students may be allowed to dramatize the purchase of five-cent pencils. First, a student buys one pencil, a second buys two, and a third buys three. They may count the money each time. If the class prefers to imagine these steps, the same results will be obtained. If a pencil is priced at five cents, one costs that much, two cost ten cents, and three cost fifteen. By counting, or observation, or experience, the students establish the fact that the formula holds true when $n=1$, or 2, or 3. Does it hold when n is increased by 1? The students count 15 cents and 5 cents making 20 cents.

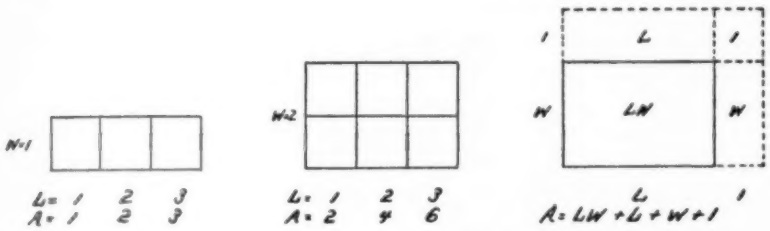
Then,	$c=15+5.$
But this is the same as,	$c=4\times5.$
And,	$4=n+1.$
Therefore,	$c=(n+1)5.$

Now, $(n+1)$ has replaced n in the original formula, and the variables have exactly the same relation as before. This means that the formula holds true for a value of n one greater than previously established. As the formula was correct when $n=3$, it is now correct when $n=4$. Having been established for $n=4$, it is established for $n=5$, then for $n=6$, and so on for all integral values of n. This type of reasoning is called Mathematical Induction.

The expression "structure of a formula" is used to mean the relation between its terms and variables as expressed in symbols.

If the structure of one forumla is the same as another, these relations are the same in both. The values of the variables may have changed so that they may be represented by different symbols, as $(n+1)$ in place of n, but if the relation remains the same, the formulas have the same structure.

The fundamental principle of Mathematical Induction is this. If a formula has been established by experiment or in any other way for small values of a variable n, and if it can be shown when the variable is increased by one that $(n+1)$ replaces n in the original formula, and its structure remains the same, then the formula is true for a value of n one greater than previously established, and so on successively, and hence for all values of n.



Let us consider another very familiar formula, $A = LW$, the formula for the area of a rectangle. Of course, this formula has been proved by the methods of geometry. Let us see whether it can be proved by Mathematical Induction. The first step is the interpretation of the formula. In more difficult formulas, this procedure tends to prevent errors. Here, obviously, the meaning is that the area of a rectangle equals the product of its length and width.

Suppose $W = 1$. Draw a square, one unit each way.

By the definition of the unit of area, $A = 1$.

Draw another square of the same size adjoining the first on the right, making a rectangle with $W = 1$, and $L = 2$. By counting squares, $A = 2$, which is the product of 1 and 2.

Draw a third square like the first joined similarly to the others making a rectangle with $W = 1$, and $L = 3$. By counting squares, $A = 3$, which is the product of 1 and 3.

The formula is now established when $W = 1$, and $L = 1$, or 2, or 3.

Suppose $W = 2$. Similarly, draw three adjacent rectangles, each two units high and one unit the other way. Then, by counting, when $L = 1$, or 2, or 3, $A = 2$, or 4, or 6. The formula is now established for $W = 1$, or 2, and $L = 1$, or 2, or 3.

Suppose L and W are each increased by 1. Draw a rectangle two units wide and three long. This is the largest figure whose area has been proved, and we may label the length L and the width W. Then using dotted lines, add a unit of length and a unit of width. The area has been increased by three figures. One is a rectangle, one unit wide and L units long. The second is a rectangle one unit wide and W units long. The third is a square one unit each way. As none of these dimensions exceed those for which the formula has been established, the total area of the whole figure can be represented by the formula, $A=LW+L+W+1$. Or, by factoring the right member, $A=(L+1)(W+1)$. Comparing this formula with the old, $(L+1)$ has replaced L, and $(W+1)$ has replaced W, while the formula has the same structure as before. Therefore the formula holds true when both L and W are increased by 1, and hence for all integral values of L and W.

In a similar way, L and W may be started as any fractions, such as $\frac{1}{2}$ and $\frac{1}{4}$, and the formula proved true for $\frac{3}{2}$ and $\frac{5}{4}$, and so on indefinitely. Thus a well known formula has been proved by Mathematical Induction.

This illustrates a different type of thinking from the proofs of geometry or the solution of equations. The purpose of using formulas from geometry and algebra was not to prove something new, or to prove an old formula in a better way, but to illustrate by the use of familiar statements just how this type of reasoning is carried on. But there are some relations which can be proved better and perhaps only by Mathematical Induction. The best examples of these are the ones concerned with the sum of a series of numbers.

For instance: $1+2+3+\cdots+n=\frac{1}{2}n(n+1)$. This means that the sum of the consecutive integers is half the product of the number of integers by one more than the number of integers. By trial, this holds true when $n=1$, or 2, or 3. If it is true when the number of terms is increased by 1, adding $(n+1)$ to both sides,

$$\begin{aligned} 1+2+3+\cdots+n+(n+1) &= \tfrac{1}{2}n(n+1)+(n+1) \\ &= \tfrac{1}{2}(n^2+n+2n+2) \\ &= \tfrac{1}{2}(n^2+3n+2) \\ &= \tfrac{1}{2}(n+1)(n+2). \end{aligned}$$

In this result $(n+1)$ has replaced n in the original formula, and the structure is the same as before. Hence the formula is true for

a value of n one greater than previously determined. As it is true for $n=3$, it is true for $n=4$, and so on indefinitely, and is true for all integers.

But in more difficult formulas, the steps of reduction on the right side are often tedious and difficult. Since the left side is unchanged throughout this reduction, the right side after the $(n+1)$ term is added equals the right side after $(n+1)$ has been substituted for n in its original form whenever the formula holds true for a value of n one greater than previously established. Then, why not make the two substitutions, and set the results equal as an hypothetical equation? If this equation can be reduced by correct steps to an identity, this hypothetical equation is true, the formula holds true for a value of n one greater than previously determined, and then, by Mathematical Induction, is established for all values of n.

If this had been done in the preceding instance:

$\frac{1}{2}n(n+1)+(n+1)=\frac{1}{2}(n+1)(n+2)$.

Dividing both sides by $(n+1)$, $\frac{1}{2}n+1=\frac{1}{2}(n+2)$.

Clearing of fractions, $n+2=n+2$.

The last equation is an identity.

The equation from which it is derived is true.

The formula is true for all values of n.

This method of proof requires fewer and easier steps than the other method.

Therefore, the following steps are necessary and sufficient to establish a proof by Mathematical Induction.

1. Interpret fully the meaning of the original formula, or write a formula from the original word statement.

2. Test the formula for values of the variable n equal to the first three integers 1, or 2, or 3. If it is correct:

3. Add the next term, the $(n+1)$ term to both sides. Do not rewrite the left side. Put down only the result of adding the $(n+1)$ term to the right side. The $(n+1)$ term is not merely $(n+1)$ but is found by substituting $(n+1)$ for n in the last term only of the series.

4. Substitute $(n+1)$ for n in the whole right side of the original formula.

5. Set the expressions found by steps 3 and 4 equal, and if possible reduce this hypothetical equation to an identity.

6. If the hypothetical equation is true, the formula is true for a value of n one greater than previously determined, and hence is true for all values of n.

The following is an example of this procedure. To prove:

$$(1)(2)+(2)(3)+(3)(4)+\cdots+(n)(n+1)=1/3(n)(n+1)(n+2).$$

1. This means that the sum of several terms, each of which is the product of the number of the term by one more than the number of the term, equals one third the product of the number of the last term by one more and by two more than that number.
2. This formula is correct for $n=1$, or 2, or 3.
3. Adding the $(n+1)$ term, the right side becomes,

$$1/3(n)(n+1)(n+2)+(n+1)(n+2).$$

4. Substituting $(n+1)$ for n in the original right side, it becomes,

$$1/3(n+1)(n+2)(n+3).$$

5. Setting these expressions equal and dividing by the common factors $(n+1)(n+2)$, $1/3(n)+1=1/3(n+3)$.
Clearing of fractions, $n+3=n+3$
6. This is an identity. Therefore, the equation set up in step 5 is true, and the formula is proved.

Students who have been taught Mathematical Induction in this way appear to have a clear understanding of the unit, and seem to enjoy this comparatively easy method of teaching and using this powerful mathematical tool.

HIGH LIGHTS OF THE MAY SKIES

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[EDITOR'S NOTE: This is the seventh of a series of articles on popular astronomy. If these articles are saved from month to month, they make a convenient handbook of astronomical facts and classroom activities for teachers in elementary and secondary school.]

With the much-heralded "April showers" about over and the "May flowers" beginning to bloom, the skies begin to show summer patterns. The days are lengthening rapidly, and the skies are becoming consistently clearer, which makes star gazing a more popular pastime. Thomas Carlyle, the English essayist once wrote "Thousands of generations, all as noisy as our own, have been swallowed up by time. . . . Yet Arcturus and Orion, Sirius and the Pleiades, are still shining in their courses, clear and

young, as when the shepherd first noted them in the plain of Shinar." And so the celestial spectacles continue undaunted by the schemes of men.

I. Constellations and Stars

Arcturus is one of the stars of the month! It can be easily located as the brilliant luminary in the southern end of *Bootes*. *Arcturus* is a huge, red hot sun about 25 million miles in diameter, and though it is one of the stars closest to the earth, its light must travel about 36 years to reach us.

MAY 1–7 TO 9 P.M. JUNE 1–9 TO 11 P.M.

Arcturus was one of the first stars proven to be moving. Its speed through space is estimated to be about 75 miles per second, and it was one of the first stars known to the earliest astronomers.

Just to the east of *Bootes* is a small semicircle of stars, known as *Corona* (Borealis). Nearly in the center of the semicircle formed by this group is the remarkable variable star *R Corona*. The star varies in brightness in a way that is unexplainable. Variable stars are quite numerous in the skies. The first one found in any given constellation is designated by the letter "R," followed by the name of the constellation. The second one discovered in the same constellation is named "S," and so on until the alphabet is exhausted, when they start again with "RR," "RS," and so on.

Many astronomers are convinced that the variable stars hold valuable clues to astronomical problems. There are now over five thousand variable stars known, and their behavior is carefully watched by professional and amateur astronomers the world over. There are variables which vary periodically and regularly, and some which seem to have no regular period. There is another group that seems to be in a continuous change. Perhaps the first class can be explained by assuming that the star is made up of two components revolving about a common center of gravity. The darker one may regularly swing in between us and its brighter companion, shutting off part of the light. Once the constellation *Corona* (Borealis) has been found, it is not hard to locate the variable with the aid of a good star map. If a search for it is made and it cannot be found, it is safe to assume that it is at a minimum period of brightness and quite beyond your vision. When at its brightest, at 6th magnitude, it can be seen with the unaided eye. It may stay at this magnitude for years, then suddenly, without any previous warning, it will begin a precipitate decrease in brightness and may drop as low as the 14th magnitude in a short time. This means that it has decreased 1585 times in luminosity.

Traveling southeast in the sky from *Bootes* near the eastern horizon is *Antares* in *Scorpius* on its way westward to become a summer luminary. It is three times farther away than *Arcturus* but appears to be nearly as bright. From *Antares* looking west along the ecliptic, *Spica* shines out in glittering radiance, marking the constellation *Virgo*, the configuration that includes more of the ecliptic than any other constellation. Further north and to the west along the ecliptic is *Regulus* in *Leo* and just to the west, sinking fast over the horizon, is *Procyon* in *Canis Minor*, making its last appearance until December comes again. North and east from *Procyon* near the horizon is *Pollux* in *Gemini* just over the ecliptic. Following the horizon northward sparkles *Capella* in *Auriga*, and from here a swift glance across the sky at the same latitude shows *Deneb* flickering in the constellation *Cygnus*. Directly above *Deneb* the brilliant star *Vega* is shining in the center of *Lyra*. And so with *Arcturus*, *Antares*, *Spica*, *Regulus*, *Procyon*, *Pollux*, *Capella*, *Deneb*, and *Vega* we have the nine celestial monarchs of May with the ever-visible *Polaris* marking the north celestial pole.

II. The Planets

In the planetary family, the "Queen of the May" is *Venus*, ruling the firmament like a brilliant jewel. On Monday, May 20, our sister planet glorifies herself in her peak of brilliance, and in the shade of a building *Venus* may be seen in the daytime.

Venus is a planet about the same size as the earth and is continually covered with a blanket of clouds. Some scientists suggest that there is more opportunity for life to exist there than on *Mars*, for *Venus* has an abundance of atmosphere to moderate the climate to compare with that of the earth, although no trace of water vapor or oxygen has been detected. It is conceivable that the conditions below the clouds covering the surface of *Venus* are quite different from conditions at the upper limits of the cloud layer. Water vapor may be present below. The temperature of the dark side of the planet has been estimated to be about 25°C. and the light side only a few degrees higher. Here again we refer to the upper atmosphere, as little is known of the actual surface of our sister planet.

Mercury, the smallest of the major planets, shows off in its brightest raiment on May 24, but, because of its size, it is not the spectacle that its neighbor *Venus* is. *Jupiter* and *Saturn* are the "morning glories" of May and are visible from dawn to sunrise and then fade in the brilliance of the

sun. *Uranus*, *Neptune*, and *Pluto* are telescopic objects and become of secondary interest to the star gazer.

III. Classroom Activities

Knowing the names of the important observatories and something about their equipment as well as the underlying principles that function in the operation and construction of telescopes are important topics for classroom investigations and discussions. It is generally thought by the casual star gazer that a telescope must have high magnification in order to see faint celestial objects. Telescopes are based on the principle of bringing light from the star to a focus where the light is viewed with an eyepiece.

The World's Largest Telescopes

Refractors	Lens Diameter	*Reflectors*	Lens Diameter
Yerkes, Williams Bay, Wis.	40″	C.I.T., Pasadena, Cal.	200″
Lick, Mt. Hamilton, Cal.	36″	Mt. Wilson, Mt. Wilson, Cal.	100″
Pittsburgh, U. of Pittsburgh	36″	Mt. Lock, Fort Davis, Tex.	82″
Meudon, France	32½″	Dunlop, Univ. of Toronto	74″
Potsdam, Germany	31½″	Dominion, Victoria, B. C.	72″
Pulkova, Russia	30″	Ohio Wesleyan, Delaware, O.	69″
Nice, France	30″	Oak Ridge, Harvard Univ.	61″
Royal, Greenwich, England	28″	Harvard, South Africa	60″
Naval, Washington, D. C.	26″	Lowell, Flagstaff, Ariz.	39″

Assembling a Classroom Telescope for Outdoor Use

There are two general types of telescopes, both capable of several variations for special purposes. The reflector gathers the light with the aid of a parabolic mirror and reflects it to a focus in front of the mirror. The re-

fractor allows the light to pass through the lense to a focus behind it, where the image is also viewed and magnified with the eyepiece. It is obvious that the larger the aperture, the greater the light gathering power. A mirror or lense may be compared to a tube placed out in the rain. The larger the diameter of the tube the more it will gather.

The magnification is generally controlled by the eyepiece. If a telescope is focused on a faint object, and not enough light is gathered by the lense to make the star visible to the eye, it can not be seen in the telescope regardless of the magnification. We would reason then, that for viewing faint stars, a telescope of generous aperture, to gather maximum light, would be desirable. Magnification is not needed in this case, for stars always appear as points of light. Comets and nebulae are likewise best seen as small, well-defined objects, such as a large aperture with low magnification will produce.

For the planets, moon and sun, a telescope with the emphasis on magnification will be more practical, for the reason that sufficient light is always available from the object itself to stand magnification.

One more point in favor of the short-focus, large-aperture telescope for general use, is that the field of view is not so restricted, and the object is easy to locate. This is a great advantage.

In conclusion, a telescope of fairly large aperture and rather moderate magnifying power would be best for school use. If more magnification is desired, an eyepiece of greater power can be fitted to it. For terrestrial use, a separate eyepiece may be purchased which will produce an erect image.

In the accompanying picture we observe a group of children gathered around a small telescope of six inch aperture made by the authors. It is one of the short focus, wide angle reflectors recommended in this article. The mounting is made from ordinary three-quarter inch pipe fittings. The tube is heavy gauge furnace pipe. With the single eyepiece it has a magnifying power of 26. Its operation is the acme of simplicity, and with an erecting eyepiece fitted to it, this small telescope makes a fine instrument for bird hunting and other terrestrial uses.

PROBLEM DEPARTMENT

CONDUCTED BY G. H. JAMISON

State Teachers College, Kirksville, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The Editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to G. H. Jamison, State Teachers College, Kirksville, Missouri.

SOLUTIONS AND PROBLEMS

NOTE. Persons sending in solutions and submitting problems for solution should observe the following instructions.

1. Drawings in India ink should be on a separate page from the solutions.

2. Give the solution to the problem which you propose if you have one and also the source and any known references to it.

3. In general when several solutions are correct, the one submitted in the best form will be used.

LATE SOLUTIONS

1639. *Walter R. Warne, Rochester, N.Y., Emmett E. Brown, Lima, N. Y. and Mrs. W. R. Warne, Rochester, N.Y.*

1645. *John P. Hoyt, Cornwall, N. Y.*

1646. *Proposed by Stephen Droenus, Willard, N. Y.*

If a, b, c, are real and different positive numbers, show that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}<\frac{a^5+b^5+c^5}{a^3b^3c^3}.$$

Solution by John P. Hoyt, Cornwall, N. Y.

Using the fundamental inequality $m^2+n^2+p^2>mn+np+mp$ where m, n, and p are unequal positive numbers, and a, b, and c are unequal positive numbers.

(1) $$a^5+b^5+c^5>a^4b^4+a^4c^4+b^4c^4$$

(2) $$a^4b^4+a^4c^4+b^4c^4>a^4b^2c^2+b^4a^2c^2+c^4a^2b^2$$

(3) $$a^4b^2c^2+b^4a^2c^2+c^4a^2b^2>a^3b^3c^2+a^3c^3b^2+b^3c^3a^2.$$

Comparing steps 1, 2, 3,

(4) $$a^5+b^5+c^5>a^3b^3c^2+a^3c^3b^2+b^3c^3a^2.$$

Dividing (4) by the positive number $a^3b^3c^3$

(5) $$\frac{a^5+b^5+c^5}{a^3b^3c^3}>\frac{1}{a}+\frac{1}{b}+\frac{1}{c}.$$

A solution was also offered by Charles W. Trigg.

1647. *Proposed by a Reader.*

To draw a circle passing through points A and B of a given circle and cutting another given circle at opposite ends of a diameter.

Solution by C. W. Trigg, Los Angeles City College

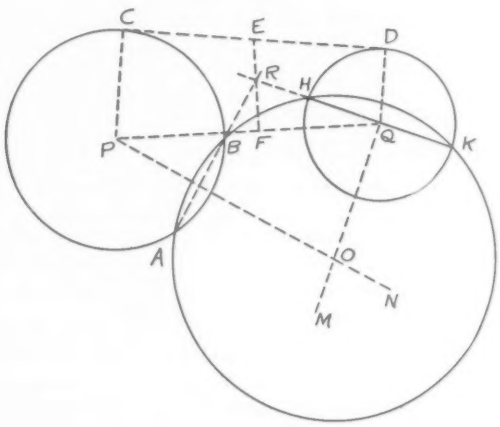
Construction.

Let the center of the circle on which A and B lie (or of any circle through A and B) be at P and let the center of the other given circle be Q. In the well-known manner construct CD the common tangent to circles P and Q. Bisect CD at E. From E drop EF perpendicular to the line of centers PQ. Draw AB extended meeting EF in R. Draw a line through R and Q meeting circle Q in H and K. Erect PN the perpendicular bisector of AB, and QM the perpendicular bisector of HK. These perpendicular bisectors will meet at O. With O as center and OA as radius describe the required circle.

Proof:

EF is the radical axis (locus of points from which the tangents to two circles are equal) of circles P and Q. AB is the radical axis of P and the

required circle. The radical axes of three circles taken two at a time are concurrent. Hence the radical axis of Q and the required circle must pass through R, and since it is required to be a diameter of circle Q it must also pass through Q. So AB and HK are chords of the required circle. Therefore the intersection of their perpendicular bisectors is the center of a circle passing through their extremities.



Solutions were also offered by George J. Ross, Brooklyn, N. Y., Fred A. Miller, Newell, W. Va., John P. Hoyt, Cornwall, N. Y., Lee A. Warne, Boulder, Colo.

1648. *Proposed by F. H. Wade, Chicago, Ill.*

Given the series

$$\sinh x = x + \frac{x}{3!} + \frac{x}{5!} + \cdots$$

$$\cosh x = 1 + \frac{x}{2!} + \frac{x}{4!} + \cdots .$$

Find the series for

$$\frac{\sinh^2 x}{\cosh x - 1} .$$

Solution by William L. Fields, Pine Bluff, Arkansas

From the fundamental identity in hyperbolic functions,

$$\cosh^2 x - \sinh^2 x = 1$$

And using the Maclaurin expansion for

$$\cosh x = 1 + \frac{x}{2!} + \frac{x}{4!} + \cdots$$

$$\frac{\sinh^2 x}{\cosh x - 1} = \cosh x + 1 = 2 + \frac{x}{2!} + \frac{x}{4!} + \cdots .$$

Solutions were also offered by George J. Ross, Brooklyn, N. Y., C. W. Trigg, Los Angeles, John F. Wagner, Chicago, Ill., Stephen F. Austin, Nacogdoches, Texas.

1649. *Proposed by Cecil B. Read, Wichita, Kansas.*

Solve

$$x+y+z=6$$
$$xy+yz+zx=11$$
$$xyz=6.$$

Solution by D. G. Buckley, Upper Canada College, Toronto, Canada

Using results from the Theory of Equations it is seen that x, y, z, are the roots of the cubic equation

$$a^3-6a^2+11a-6=0. \tag{1}$$

Factoring (1),

$$(a-1)(a-2)(a-3)=0.$$

Therefore

$$a=1, 2, 3.$$

Hence from the symmetry of the equations,

$$(x, y, z)=(1, 2, 3); (1, 3, 2); (2, 3, 1); (2, 1, 3); (3, 1, 2); (3, 2, 1).$$

Solutions were also offered by Stephen F. Austin, Nacogdoches, Texas, John F. Wagner, Chicago, Ill., B. Felix John, Pittsburgh, Pa., Magaret Joseph, Milwaukee, Wis., C. E. Jerikins, Oak Park, Ill., George Crenshaw, Jr., La Fayette, Kentucky, John Hoyt, Cornwall, N. Y., Walter R. Warne, Rochester, N. Y., Lillian C. Spring, Upper Montclair, N. J., June Higgins, Carbondale, Pa., George J. Ross, Brooklyn, N. Y., Aaron Buchman, Buffalo, N. Y., Stine Jensen, Antigo, Wis., John L. Macchi, Unio City, N. J., Henry Hausdorff, Jersey City, N. J., Mozell Simpson, Norman, Okla., William Louis Fields, Pine Bluff, Ark., Charles W. Trigg, Los Angeles, Arthur Danzl, Collegeville, Minn., Howard C. Robinson, Oakdale, R.D., Pa., Roy Wild, Kirksville, Mo., Fred A. Miller, Newell, W. Va., Alan Wayne, New York, N. Y., M. Kirk, West Chester, Pa., and also by the proposer.

1650. *Proposed by Roy MacKay, Portales, New Mexico.*

In the triangle ABC, H is taken $1/n$th of the way from B to C; K is taken $1/n$th of the way from C to A; L, $1/n$th of the way from A to B. The lines AH, BK, CL form a triangle whose area is $(n-2)^2/(n^2-n+1)$ times the area of the triangle ABC.

Solution by George J. Ross, Brooklyn, N. Y.

Using the Theorem, two triangles having an angle of one equal to an angle of the other are to each other as the product of the sides including the common angle, one obtains:

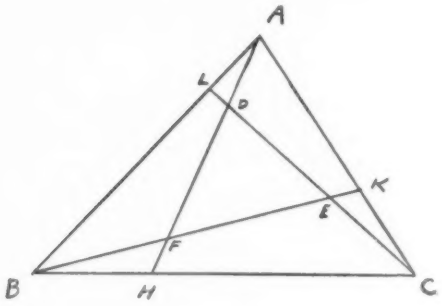
$$\frac{\triangle ALC}{\triangle ABC}=\frac{AL}{AB}=\frac{1}{n} \tag{1}$$

$$\frac{\triangle KEC}{\triangle ALC}=\frac{KC\cdot EC}{AC\cdot LC}=\frac{1}{n}\cdot\frac{EC}{LC} \tag{2}$$

$$\frac{\triangle DEF}{\triangle KEC}=\frac{DE\cdot EF}{KE\cdot EC}. \tag{3}$$

Multiply (1), (2), (3)

(4) $$\frac{\triangle DEF}{\triangle ABC}=\frac{1}{n^2}\cdot\frac{DE}{LC}\cdot\frac{EF}{KE}.$$



Now apply Menelaus's theorem to $\triangle BCL$ using ADH as the straight line cutting the sides of the triangle.

$$\frac{BH}{CH}\cdot\frac{CD}{LD}\cdot\frac{LA}{BA}=1$$

$$\therefore\quad \frac{1}{n-1}\cdot\frac{CD}{LD}\cdot\frac{1}{n}=1$$

$$\therefore\quad \frac{LD}{CD}=\frac{1}{n(n-1)}.$$

(5) By composition $$\frac{LD}{LC}=\frac{1}{n^2-n+1}.$$

Apply Menelaus's theorm to KEB cutting $\triangle ALC$, and using the results of (5)

(6) $$\frac{CE}{LC}=\frac{n}{n^2-n+1}.$$

Adding (5) and (6)

$$\frac{LD+CE}{LC}=\frac{n+1}{n^2-n+1}.$$

(7) By division $$\frac{DE}{LC}=\frac{n^2-2n}{n^2-n+1}.$$

Apply Menelaus's theorm to LEC cutting $\triangle BKA$, and one obtains as in (5)

(8) $$\frac{KE}{BK}=\frac{1}{n^2-n+1}.$$

Apply Menelaus's theorm to AFH cutting $\triangle BKC$ and one obtains as in (6) by composition

(9) $$\frac{BF}{BK}=\frac{n}{n^2-n+1}.$$

Add (8) and (9)

$$\frac{BF+KE}{BK}=\frac{n+1}{n^2-n+1}.$$

(10) By division
$$\frac{EF}{BK}=\frac{n^2-2n}{n^2-n+1}.$$

Divide (10) by (8)

(11)
$$\frac{EF}{KE}=n^2-2n.$$

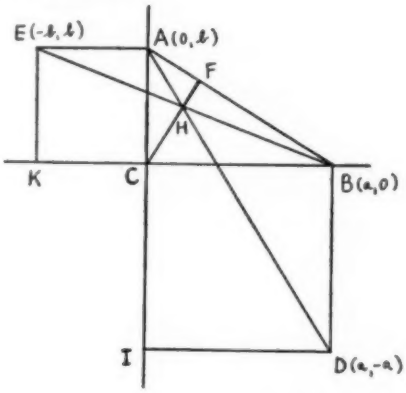
Substitute (7) and (11) in (4)

$$\frac{\triangle DEF}{\triangle ABC}=\frac{(n-2)^2}{n^2-n+1}.$$

Solutions were also offered by Orville W. Brown, Canuga, N. Y., Mrs. Walter R. Warne, Rochester, N. Y., John Kilpatrick, Nacogdoches, Texas, C. W. Trigg, Los Angeles, P. D. Thomas, B. Felix John, Pittsburgh, Pa., M. Kirk, West Chester, Pa., Aaron Buchman, Buffalo, N. Y., Walter R. Warne, Rochester, N. Y.

1651. *Proposed by John P. Hoyt, Cornwall, N. Y.*

If squares $BDIC$ and $ACKE$ are constructed on legs BC and AC respectively of right triangle ABC, the lines AD and BE are concurrent with the altitude to the hypotenuse.



Solution by Aaron Buchman, Buffalo, N. Y.

Take triangle ACB with C at the origin of the rectangular coordinates, CB along the X axis, and AC along the Y axis. Let B have the coordinates $(a, 0)$, and let A have the coordinates $(0, b)$. Then the coordinates of D are $(a, -a)$ and the coordinates of E are $(-b, b)$.

The equation of CF, the altitude to side AB, is

(1)
$$y/x=a/b.$$

The equation of the line through A and D is

(2)
$$(y-b)/x=(b+a)/(-a).$$

The equation of the line through B and E is

(3)
$$y/(x-a)-(-b)/(a+b).$$

Solving (2) and (3) simultaneously, the coordinates of their intersection H are

$$\left(\frac{ab^2}{a^2+ab+b^2}, \frac{a^2b}{a^2+ab+b^2}\right).$$

Since the coordinates of H, by inspection, satisfy equation (1), then CF, AD, and EB are concurrent.

Some solutions similar to this and others using the converse of Ceva's Theorem were offered by: James A. Lemon, Eaton, Ohio, George J. Ross, Brooklyn, N. Y., C. W. Trigg, Los Angeles, Walter R. Warne, Rochester, N. Y., Fred A. Miller, M. Kirk, West Chester, Pa., and also by the proposer.

HIGH SCHOOL HONOR ROLL

The Editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

For this issue the Honor Roll appears below:

1647. *Sidney V. Soanes, Toronto, Ontario, Canada.*

1649. *Kenneth P. Carlson, Dannebrog, Nebraska, Edward Rosendal, Dannebrog, Nebraska, John R. Sizelove, Eaton, Ohio, David Ker, Toronto, Canada, R. W. L. Lardlaw, Toronto, Canada, Sidney V. Soanes, Toronto, Canada.*

1651. *D. G. Buckley, Toronto, R. W. L. Lardlaw, Toronto, Sidney V. Soanes, Toronto.*

PROPOSED FOR SOLUTION

1664. *Proposed by Harry Frye, Tallahoma, Tenn.*

If the sum of 636 consecutive odd numbers is a cube, what are the numbers.

1665. *Proposed by C. W. Trigg, Los Angeles City College.*

The orthogonal projections of any point in the plane of a triangle upon the altitudes of the triangle are the vertices of a triangle similar to the given triangle.

1666. *Proposed by Cecil B. Read, Wichita, Kan.*

If ABC and ABD are equilateral triangles in perpendicular planes, find the angle CAD.

1667. *Proposed by Cecil C. Read, Wichita, Kan.*

If $2x$ is a positive acute angle show that $\cot x - \cot 4x > 2$.

1668. *Proposed by B. Felix John, Pittsburgh, Pa.*

A plane parallel to the base of a pyramid divides the lateral area into equal parts. In what ratio does the plane cut the altitude?

1669. *Proposed by John P. Hoyt, Cornwall, N. Y.*

If three edges of a tetrahedron, of lengths a, b, and c, are mutually perpendicular and concurrent at one vertex of the tetrahedron, prove (by

elementary geometry) that the remaining altitude of the tetrahedron is equal to

$$\frac{abc}{\sqrt{a^2b^2+a^2c^2+b^2c^2}}.$$

SCIENCE QUESTIONS

May, 1940

Conducted by Franklin T. Jones,
10109 Wilbur Avenue, S.E., Cleveland, Ohio

DO YOU KNOW THE ANSWERS?

91. Can X-rays go through black paper?
92. Is ocean water more salty today than it was a century ago?
93. Is a "tidal wave" caused by the action of the tides?
94. Is a sponge a plant or an animal?
95. Has any man ever gone to a depth of four miles in the ocean?

Answers to March Questions

81. The 1940 decade begins January 1, 1941.
92. Either the Sperm whale, or the Killer whale could have swallowed Jonah because either has a gullet large enough to take in "a man or even a boat." (*National Geographic Magazine*, Jan. 1940, p. 35.)
83. A neon sign glows red not blue so the statement, "John Nance Garner's *neon-blue* eyes is just 'newspaper science.'"
84. "The secret of the new high-test safety glass . . . is a new plastic called polyvinyl acetal resin. The thin sheets of this plastic, which are sandwiched between glass plates, look and act like transparent rubber. They are springy enough to come back to shape after stretching and keep their elasticity even when cold. Under heat and pressure the plastic makes a permanent bond with the glass without any other adhesive, producing a safety glass that is so tough that a half-pound weight dropped a height of 84 feet will not shatter a quarter-inch pane." (*Life*, January 1, 1940, p. 32.)
85. A small stone will go to the bottom of the ocean in any depth because stone is specifically heavier than water at any depth.

WANTED

QUESTIONS—TESTS—EXAMINATION PAPERS—ANSWERS

Questions your pupils should answer correctly but do not.
Questions that aroused discussion in your classes.
The fun as well as the serious in science teaching.

Join the Guild of Question Raisers and Answerers (GQRA)
Pupils and classes are eligible as well as teachers and outsiders.
332 are already members!

GQRA—NEW MEMBERS, May, 1940

333. *Benjamin De Leon, South Side High School, Newark, N. J.*
334. *Walter Hladky, student, Newark College of Engineering, Newark, N. J.*
335. *Rosalyn Heck, Mercy High School, Milwaukee, Wis.*
336. *L. Grossweiner, Bronx High School of Science, New York.*
337. *Arthur Berman, Bronx HS of Science, New York City.*

MAGNETIC INTENSITY

889. *Proposed by L. Grossweiner, Bronx H.S. of Science, New York City. (Elected to the GQRA, No. 336.)*

"We explain by the electron theory, in a magnet, all poles of each molecule are opposite, hence they tend to neutralize the magnetism except at the magnetic poles, where magnetism is apparent.

"Why, therefore, does magnetism slowly increase from the center to the ends of a magnet instead of being apparent only at the poles?"

---

890. *Proposed by Arthur Berman, Bronx H.S. of Science, New York City. (Elected to the GQRA, No. 337.)*

"How can astronomers find out whether a star is moving directly toward us or away from us and even calculate its speed?"

---

891. *Try your Kinetic Energy formula on this and interpret.*

It is said that a quarter-pound weight dropped from a height of 84 feet will not shatter a quarter-inch pane of the new safety glass manufactured by Libby-Owens-Ford.

Could a man put his fist through this pane of glass?

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892. *Part of a Physics Test submitted by C. S. Greenwood (GQRA No. 135) Sheffield, Pa.*

PHYSICS #2 JANUARY 1940 NAME_____

C. Write in each blank one or more words to complete the sentence:

1. The water level in the spout of a tea kettle is _____ that in the body of the kettle.
2. A liquid exerts an upward force on a given surface which is _____ _____ the downward force on an equal surface at the same depth.
3. The number of units of weight per one unit of volume is called _____.
4. Bodies submerged in a liquid _____ weight equal to the weight of the _____.
5. The number which expresses the relation between the weight of an equal volume of water is called its _____.
6. The _____ is a device for testing the specific gravity of liquids.
7. Pressure applied to a confined liquid is _____ equally in all _____ and acts with equal force on all _____ surfaces.
8. The liquid force on the bottom of a cylindrical vessel containing water to a certain height is _____ that on the bottom of a conical vessel having an equal base and containing water to the same height.

9. A floating body sinks until the liquid it displaces is equal to _____

_____.

Collision

872. *Proposed by Shirly Sporcic (GQRA, No. 325), Normandy H. S., St. Louis, Mo.*

Suppose that a ball, A, weighing $3x$ grams, is suspended in a stationary position. Another ball, B, weighing x grams (that is A is three times as heavy as B), is suspended from the same height. The latter, B, is drawn back and then allowed to strike the stationary sphere.

Now, the question is, what will happen to each one of the balls? (We intend to determine the answer by experiment.)

Answer by John Kilpatrick (GQRA No. 250), Stephen F. Austin, State Teachers College, Nacogdoches, Texas.

I haven't seen a solution to your problem 872 yet. This is a general solution to the problem of bodies colliding in a straight line.

Let v_1 be velocity of mass m before collision and u_1 afterwards. and v_2 be velocity of mass M before collision and u_2 afterwards.

The total momentum is not changed:

$$mu_1+Mu_2=mv_1+Mv_2. \tag{1}$$

Newton found that the difference of the velocities of two bodies before and after collision has a constant ratio. (This constant is a measure of the elasticity of the bodies.)

$$u_1-u_2=k(v_1-v_2). \tag{2}$$

Solving,

$$\begin{aligned} &mu_1+Mu_2=mv_1+Mv_2\\ &\underline{Mu_1-Mu_2=kM(v_1-v_2)}\\ &(m+M)u_1=mv_1+Mv_2+kM(v_1-v_2)\\ &u_1=\frac{mv_1+Mv_2}{m+M}+k(v_1-v_2)\,\frac{M}{M+m}. \end{aligned}$$

In the same way,

$$u_2=\frac{mv_1+Mv_2}{m+M}-k(v_1-v_2)\,\frac{m}{M+m}.$$

In this problem, body A has mass $m=3x$, $v_1=0$, B has mass $M=x$, v_2 not stated.

$$u_1=\frac{v_2x}{4x}+k(-v_2)\,\frac{x}{4x}=v_2\,\frac{1-k}{4}\ \text{(velocity of } A \text{ after impact)},$$

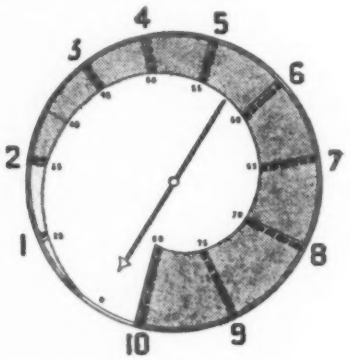
$$u^2=\frac{v_2x}{4x}-k(-v_2)\,\frac{x}{4x}=v_2\,\frac{1+3k}{4}\ \text{(velocity of } B \text{ after impact)},$$

k as we have said measures the elasticity of the two bodies. If $k=0$ they are completely inelastic, very much like soft tar. If $k=-1$ they are perfectly elastic, like good steel.

If the balls A and B are perfectly elastic, A will move in the positive direction (direction in which B fell) with velocity $v_2/2$, B in the opposite direction with the same velocity.

TURNABILITY AND DANGER UNITS

881. *Suggested by a booklet published by one of the great insurance companies on the subject of SPEED and DANGER.*



The cut shows a new kind of meter—a sort of ENERGY METER.

The heavy figures on the outside of the circle are DANGER UNITS.

1 Danger Unit equals one ROLL-OVER.

The lighter figures on the inside of the helix are miles per hour (MPH).

Paste this meter on the dash of your car and think as you drive—"Am I carrying too many roll-overs?"

Then we shall think, in terms of energy which reflects DANGER.

When you are *rolling along*, think about *rolling over*.

At 25 miles per hour (MPH), a moving car has developed about enough energy to roll over once. Your body could probably *take* this.

At 50 MPH, it has developed not twice but four times the energy—enough to roll over *four times*. You'd have to be lucky to come out alive.

At 75 MPH, it has developed not three times but nine times the energy or enough to roll over *nine times*. Of course, it would hit a tree or a culvert long before it did its ninth somersault and then the unused portion of the energy would be spent in ripping the culvert, the car and the occupants to pieces.

THINK ABOUT YOUR TURN OVERS AS YOU DRIVE.

ACTIVITY OF COPPER AND ZINC

866. *Proposed by Lawrence Corey, Choteau, Montana (GQRA No. 294).*

"How is it that copper is so much farther down in the electrochemical series than zinc and yet its valence is between one and two, while zinc is always two; furthermore, the International atomic weight of zinc is 65.37 and the atomic weight of copper is only 63.57? These are, I believe, the two main factors which determine the activity of metals and they both seem to show that zinc should be more active."

I don't know myself and so I have quizzed around among my learned professorial friends. The net result seems to boil down to about this:—

"There is no mathematical relation between position in the electrochemical series and the periodic table or valence."

The consensus of opinion seems that this is "just one of those things that is so."

Perhaps you have a good or better answer. Please send it along.

Write again and send in some more questions.

Get some of your pupils to answer some of the questions and so get into the *GQRA*. The boys and girls seem to like to see their names in print.

BOOKS AND PAMPHLETS RECEIVED

REFERENCE BOOK OF INORGANIC CHEMISTRY, by Wendell M. Latimer, *Professor of Chemistry in the University of California*, and Joel H. Hildebrand, *Professor of Chemistry in the University of California*. Revised Edition. Cloth. Pages xii+563. 14×21 cm. 1940. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $4.00.

EXPERIMENTING AT HOME WITH THE WONDERS OF SCIENCE, by Eugene Hodgdon Lord, *Teacher of Physics and General Science, Boston Latin School, Boston, Massachusetts*. Cloth. Pages xii+243. 13×19.5 cm. 1940. D. Appleton-Century Company, 35 West 32nd Street, New York, N. Y. Price $2.00.

SCIENCE WITH SIMPLE THINGS, by Raymond F. Yates, *Author of "Exploring with the Microscope," "How to Make Electric Toys,"* etc. Cloth. Pages xv+245. 13.5×20.5 cm. 1940. D. Appleton-Century Company, 35 West 32nd Street, New York, N. Y. Price $2.50.

SEMIMICRO QUALITATIVE ANALYSIS, by William Lloyd Evans, Alfred Benjamin Garrett, and Laurence Larkin Quill, *The Ohio State University*. Cloth. 246 pages. 19.5×26.5 cm. 1940. Ginn and Company, 15 Ashburton Place, Boston, Mass. Price $2.00.

MATHEMATICS REVIEW EXERCISES, by David P. Smith, Jr., and Leslie T. Fagan, *Both Masters in Mathematics in the Lawrenceville School, New Jersey*. Cloth. Pages vii+280. 14.5×23 cm. 1940. Ginn and Company, 15 Ashburton Place, Boston, Mass. Price $1.28.

LIVING SPECIMENS IN THE SCHOOL LABORATORY, published by General Biological Supply House. Paper. 93 pages. 15×23 cm. March 1940. General Biological Supply House, Inc., 761–763 East 69th Place, Chicago, Ill. Price $1.00.

LEARNING TO COMPUTE, BOOKS ONE AND TWO, by Raleigh Schloring, *The University High School, University of Michigan;* John R. Clark, *The Lincoln School, Teachers College, Columbia University;* Mary A. Potter, *Supervisor of Mathematics, Racine, Wisconsin;* and Carroll F. Deady, *Superintendent of Schools, Archdiocese of Detroit*. Paper. Pages viii+104. 15.5×25 cm. 1940. World Book Company, Yonkers-on-Hudson, New York, Price each 32 cents.

THE RATE OF CHEMICAL REACTIONS, A GUIDE FOR USE WITH THE INSTRUCTIONAL FILMS "VELOCITY OF CHEMICAL REACTIONS" AND "CATALYSIS," prepared by Melvin Brodshaug, *Erpi Classroom Films, Inc.*, and W. Lyle Brewer, *Advanced School of Education, Teachers College, Columbia University*, in Collaboration with Hermann I. Schlesinger and Warren C. Johnson, *The University of Chicago*. Paper. Pages iv+35. 13.5×20 cm. 1940. The University of Chicago Press, Chicago, Ill. Price 15 cents.

EXERCISES IN REASONING, by Joseph A. Nyberg, *Hyde Park High School, Chicago, Illinois*. Paper. 84 pages. 21×28 cm. 1940. Joseph A. Nyberg, 10505 S. Bell Avenue, Chicago, Ill. Single copies 50 cents, 10 or more 30 cents each.

BOOK REVIEWS

ADVANCED CALCULUS, by I. S. Sokolnikoff, *Associate Professor of Mathematics, University of Wisconsin.* Cloth. Pages x+466. 15×23 cm. 1939. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York, New York. Price $4.00.

Here is a book on calculus, written with all the care of a professional mathematician, but which nevertheless presents the subject with a sympathetic understanding of the needs and limitations of the average undergraduate.

The first chapter on Limits and Continuity sets the tone of the entire book. First, the notion of sequence, which frequently proves a stumbling block to the student, is carefully and vividly explained. Then, after defining null sequences, the author introduces limits, the fundamental theorem on convergence, and the definitions of continuity and uniform continuity.

The remaining chapters contain discussions on Functions of Many Variable, Definite, Line, Multiple and Improper Integrals, Infinite Series, Power Series and their Applications, Fourier Series and finally a proof of the Existence Theorem for Implicit Functions.

The book is well written and can be recommended as a text for a second year course in calculus. In fact, a good student should not have much difficulty in studying the book without a teacher.

However there are two minor points which the author might consider for his next edition: the inclusion of a small table of derivatives and integrals, and some remarks about Stirling's formula for the factorial.

BERNARD FRIEDMAN
Woodrow Wilson Junior College

ADVANCED ALGEBRA, by S. Barnard, *Formerly Assistant Master at Rugby School, Late Fellow and Lecturer at Emmanuel College, Cambridge,* and J. M. Child, *Formerly Lecturer in Mathematics in the University of Manchester, Late Head of Mathematical Department, Technical College, Derby.* Cloth. Pages x+280. 13.5×21.5 cm. 1939. The Macmillan Company, 60 Fifth Avenue, New York, New York. Price $2.50.

Despite the title, this book could not be used in any secondary school and in very few colleges. The authors state explicitly that this book is intended for the mathematical specialist working for an Honors degree at the Universities.

This work covers a great deal of ground. Starting with Homographic Transformations and Cross Ratio, the authors discuss Invariants and Covariants, Infinite Series, the Law of Quadratic Reciprocity, Complex Variables and wind up with the General Theory of Continued Fractions.

As with most English text books a great many problems are included covering all parts of the subjects treated. But because of the multitude of topics, little can be done with any of them. As a result, the book seems to emphasize manipulative skill instead of comprehension. The emphasis should be on the broad inter-relations between the different branches of mathematics and not on the technical details of all of them.

BERNARD FRIEDMAN
Woodrow Wilson Junior College

Handbook of Nature-Study, by Anna Botsford Comstock, *Late Professor of Nature-Study in Cornell University.* Twenty-fourth Edition. Cloth. Pages xx+937. 17.5×26 cm. 1939. Comstock Publishing Company, Inc., 124 Roberts Place, Ithaca, N. Y. Price $4.00.

This book, revised with errors corrected and new material added, will be invaluable to both student and the teacher of average training. For the

student the author's charm of style in writing, the individualism given to the study of each topic and its surroundings, the simple language used in discussion, and the ample and distinct illustrations will have definite appeal. For the untrained teacher, for whom the author confesses to have written, the clearly stated objectives, leading thoughts, methods for observations and achievement questions will be greatly appreciated. The author's enthusiastic approach to the study of nature is contagious and the student's desire to learn is unconsciously spurred. The wealth of material with a thorough treatise of each subject is grouped into four parts; namely: (1) The Teaching of Nature Study; (2) Animals; (3) Plants; (4) Earth and Sky. The bibliography is comprehensive and is deserving of praise for both scope of content and arrangement. The volume is large but the material is all usable for elementary purpose.

MARIE E. SHIELDS
Chicago, Ill.

INTRODUCTORY COLLEGE CHEMISTRY, by Harry N. Holmes, Ph.D. *Professor of Chemistry in Oberlin College.* Third Edition. Cloth. Pages v+619. 22×15 cm. 1939. The Macmillan Company, New York. Price $3.50.

At the beginning of Chapter I "Basic Principles" the author points out how civilization rests upon chemical foundation and this should stimulate an interest in chemistry. At the end of Chapter V there is a Chapter Outline and Review Suggestions which are very helpful to students.

This edition is up to date in every respect. The conception of covalence and electrovalence is used often. The discussion of the atoms is introduced in Chapter III and is repeated frequently. A brief and good introduction is given to the Bronsted and Debye-Huckel theories. The author includes a short chapter on nuclear chemistry and one on radiation chemistry.

Discussions on industrial application are up to date. "Colloid chemistry s given rather full treatment as an extension to the chapter on solutions." The discussion on vitamins has been revised. References are at the end of each chapter. This is an excellent textbook for college classes in general chemistry for students with or without previous training in chemistry. Students can read this book and understand what they are reading.

E. G. MARSHALL

AN INTRODUCTION TO CHEMICAL SCIENCE, by W. H. Hatcher, *McGill University.* Cloth. Pages iii+410. 22×15 cm. 1940. John Wiley and Sons, Inc., New York.

The author states that the subject matter presented in this textbook is not to take the place of a course in general chemistry. It is intended for use in a non-specialist course for liberal arts students. This is an excellent textbook for use in a survey course in chemistry.

The material is divided into four main parts: inorganic, organic, food and industrial chemistry. Topics which are discussed in elementary courses are taken up in these four divisions. "The arrangement of those topics which are of fundamental importance to chemistry as a whole has been designed to show progressive treatment in successive chapters. Examples of this arrangement are to be found in the consideration of water and its solutions, the atmosphere and its compounds, carbon and its life processes and the comparative classification of the elements."

Most of the chapters are followed by a summary and carefully chosen questions. Only the fundamental laws and theories are discussed and no undue stress is laid on balancing equations and mathematical calculations.

E. G. MARSHALL

THE BOOK OF DIAMONDS, by J. Willard Hershey, M.S., Ph.D., *Department of Chemistry, McPherson, Kansas.* Cloth. Pages xii+142. 15×23.5 cm. 1940. Hearthside Press. The Chemical Publishing Co., Inc., 148 Lafayette Street, New York, N. Y. Price $2.00.

This book is intended for the general reader interested in all kinds of information about diamonds. Their properties, where found, descriptions of the mines, tests for diamonds, their value, methods of cutting, superstitions about diamonds, and other topics are discussed. The three great diamond producing regions—India, Brazil, and South Africa—are described and compared. A description and history of the famous diamonds and a short section on other gems are included. The book closes with a chapter on the various attempts to produce synthetic diamonds, principally by the author and his students at McPherson College.

The book is instructive and entertaining but clumsy sentence structure, much repetition, and lack of coherence distract attention from the content and make reading difficult.

G. W. W.

NEW AND REVISED TEXTBOOKS

SENIOR MATHEMATICS FOR HIGH SCHOOLS, by Virgil S. Mallory, *Professor of Mathematics and Instructor in the College High School, State Teachers College, Montclair, N. J.*, and Howard F. Fehr, *Assistant Professor of Mathematics and Instructor in the College High School, State Teachers College, Montclair, N. J.* Cloth. Pages ix+442. 12.5×9.5 cm. 1940. Benj. H. Sanborn & Co., 221 E. Twentieth St., Chicago, Ill. Price $1.96.

A textbook for high school seniors built around the function concept as the unifying element. If all college freshmen could enter the mathematics and physics classes with a mastery of the subject matter of this text, what a load would be lifted from the shoulders of the college teachers and what progress could be made!

ELEMENTARY ALGEBRA, by Aaron Freilich, *Chairman, Department of Mathematics, Lafayette High School, New York City*, Henry H. Shanholt, *Chairman, Department of Mathematics, Abraham Lincoln High School, New York City*, Joel S. Georges, *Chairman, Department of Mathematics, Wright Junior College, Chicago.* Cloth. Pages iv+544+25. 13×20 cm. 1940. Silver Burdett Company, 45 East 17th Street, New York, N. Y. Price $1.36.

The authors have attempted to write a book that can be read by the students; that includes much developmental material; that provides for individual differences in ability and interests; that develops power in problem solving and independent thinking.

EVERYDAY ARITHMETIC FOR PRINTERS, by John E. Mansfield, *Head of the Department of Printing, Wentworth Institute, Boston.* Cloth. Pages iii+135. 14×20 cm. McGraw-Hill Book Co., Inc., 330 W. 42nd St., New York, N. Y. Price $1.50.

A book for students of printing where arithmetic forms a part of the printing course, and for apprentices in printing.

ELEMENTARY COLLEGE MATHEMATICS, by Ernest Lloyd Mackie, Ph.D., *Professor of Mathematics*, and Vinton Asbury Hoyle, Ph.D., *The University of North Carolina, Chapel Hill, North Carolina.* Cloth. Pages ix+331+78. 14×23 cm. 1940. Ginn and Company, 15 Ashburton Place, Boston, Mass. Price $2.80.

The subject matter is principally algebra and trigonometry with a brief introduction to analytics and the calculus. The book is a mean between the old traditional four subject course and the unified course.

REVIEWS AND EXAMINATIONS IN ALGEBRA, by Oswald Tower and Winfield M. Sides, *Department of Mathematics, Phillips Academy, Andover, Mass.* Cloth. Pages v+175. 13×21 cm. D. C. Heath & Company, 285 Columbus Avenue, Boston, Mass. Price $1.20.

A review text for students who have had at least a year of high school algebra. Preparation for various college entrance examinations is prominent although the authors state that this is not the primary purpose of the book.

SCIENCE IN OUR MODERN WORLD, a three-book course for the junior high school, by Ralph K. Watkins, *Professor of Education, University of Missouri*, and Winifred Perry, *Teacher of General Science, Roosevelt Junior High School, San Diego, California.* Published 1940, The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Book I, *Understanding Science*, Cloth. Pages xiv+432. 14×21 cm. Price $1.28. Book II, *Science for Daily Use.* Cloth. Pages xii+500. 14×21 cm. Price $1.48. Book III, *Science for Human Control.* Cloth. Pages xvi+588. 14×21 cm. Price $1.68.

This series should be examined by teachers and supervisors before the text for the science classes is selected. It is attractive, covers the subject well and has many aids for pupils.

PRINCIPLES OF CHEMISTRY, by Joel H. Hildebrand, *Professor of Chemistry in the University of California.* Fourth Edition. Cloth. Pages xi+359. 14×21.5 cm. 1940. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $2.50.

This revision of the well known text was made in order to include new methods of presentation and to bring parts of the text abreast with recent discoveries and theories in physics and chemistry.

A COURSE IN GENERAL CHEMISTRY, by William C. Bray and Wendell M. Latimer, *Professors of Chemistry in the University of California.* Third Edition. Cloth. Pages xi+206. 14×21.5 cm. 1940. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $1.50.

This is a laboratory manual and study guide to accompany Hildebrand's *Principles of Chemistry* and Latimer and Hildebrand's *Reference Book of Inorganic Chemistry.*

PROBLEMS IN MECHANICS, by G. B. Karelitz, *Professor of Mechanical Engineering, Columbia University;* J. Ormondroyd, *Professor of Engineering Mechanics, University of Michigan;* and J. M. Garrelts, *Associate Professor of Civil Engineering, Columbia University.* Cloth. Pages ix+271. 14.5×23 cm. 1939. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $2.50.

This book consists of fifty pages devoted to the principles of mechanics followed by 782 problems, some of which are solved. Many diagrams are included and the answers accompany the problems. This text is of the practical type: exposition of the principles and the theorems of mechanics is given in brief form; applications are illustrated by many engineering problems.

POPULAR SCIENCE BOOKS

YOUNG CHEMISTS AND GREAT DISCOVERIES, by James Kendall, *Professor of Chemistry in the University of Edinburgh, General Secretary of the Royal Society of Edinburgh.* Cloth. Pages xvi+272. 12×19.5 cm. 1939. D. Appleton-Century Company, 35 West 32nd Street, New York, N. Y. Price $2.75.

A book describing the early life and struggles of a number of brilliant young chemists. It is not merely a book of biography. A great scientist and author shows how epoch-making experiments have influenced the progress of science and civilization. Don't miss it.

A SHORT HISTORY OF SCIENCE, by W. T. Sedgwick and H. W. Tyler. Revised by H. W. Tyler and R. P. Bigelow. Cloth. Pages xxi+512 13.5×21.5 cm. 1939. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $3.75.

A very brief account of the development of scientific knowledge from the dawn of civilization to modern times. The early and classical periods are adequately discussed for the general reader but practically nothing is given of the multitude of startling discoveries of the twentieth century.

EXCURSIONS IN SCIENCE, edited by Neil B. Reynolds, *General Electric Company,* and Ellis L. Manning, *Supervisor of Science, New York State Department of Education.* Cloth. Pages xiii+307. 13.5×20.5 cm. 1939. McGraw-Hill Book Company, 330 West 42nd Street, New York, N.Y. Price $2.50.

Thirty-five short essays by noted scientists. The subjects include archeology, odors, electron optics, animal light, earthquakes, heredity. A brief biographical sketch of the author of each article is given. It will interest young and old.

MATHEMATICAL RECREATIONS AND ESSAYS, by W. W. Rouse Ball, *Late Fellow of Trinity College, Cambridge,* and revised by H. S. M. Coxeter, *Past Fellow of Trinity College, Cambridge.* Eleventh Edition. Cloth. Pages xvi+418. 13×20 cm. 1939. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $2.75.

A book famous for nearly half a century again brought up to date by adding discoveries since the tenth revision in 1922. A host of ideas for the mathematics club.

THE PAGEANT OF ELECTRICITY, by Alfred P. Morgan. Illustrated. Cloth. Pages xxvi+363. 14×21.5 cm. 1939. D. Appleton-Century Company, 35 W. 32nd Street, New York, N. Y. Price $3.50.

Mr. Morgan, noted electrical engineer, inventor, manufacturer, writer, and research worker writes a marvelous story of the entire development of electricity. He has done the experiments, developed the apparatus and machinery for both civil and military use, and knows personally many of the men whose lives and products furnish the subject matter of his book.

NUMBER: THE LANGUAGE OF SCIENCE, by Tobias Dantzig, *Professor of Mathematics, University of Maryland.* Third Edition, Revised and Augmented. Cloth. Pages x+320. 14×21.5 cm. 1939. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $3.00.

"A book on the cultural aspects of mathematics written for the cultured non-mathematician." This edition is, with only a few minor changes, the second edition (1922) with twenty-six short articles appended, which very materially add to the interest and value of the book.

THE MARCH OF MIND, A SHORT HISTORY OF SCIENCE, by F. Sherwood Taylor. Cloth. Pages xiv+320. 14×21.5 cm. 1939. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $3.00.

This book is not a history of science in the usual meaning because it is not designed as a book of reference but rather to show a change in the attitude toward science.

BEYOND YONDER, by Oliver Justin Lee, *Director of the Dearborn Observatory, Northwestern University*. Cloth. 169 pages. 13×19 cm. 1939. Chapman and Grimes, Inc., 110 Mount Vernon Street, Boston, Mass. Price $2.50.

This book explains how scientists measure the exceedingly small objects they study, the very large ones, and the far distant ones. A book that will satisfy the curiosity of many who have asked, "How do you know or learn about such things?"

ENGINEERING'S PART IN THE DEVELOPMENT OF CIVILIZATION, by Dugald C. Jackson. Cloth. 114 pages. 13×20 cm. 1939. The American Society of Mechanical Engineers, 29 West 39th Street, New York, N. Y. Price $1.55.

A series of six lectures by one of America's great teachers. No engineering student should miss them.

PYORRHEA CAN BE PREVENTED

A bright outlook for prevention of pyorrhea and other diseases of the gums was given at the Baltimore Dental Centenary by Dr. Olin Kirkland, Montgomery, Ala. The keynote here is to be on the alert for early symptoms and treat them promptly.

"Of all the preventable diseases, there is no other that can be prevented so easily, or with any greater assurance of success. It is one of the simplest of all diseases to treat while in the early stages, and yet, it is one of the most difficult to cure when it has reached the advanced stage of development."

EARTH'S ROCKS TELL AGE WITH HELIUM GAS AS TIME CLUE

The ages of the earth's rocks, determined from the amount of helium gas present in them, can be correctly told only if geologists interpret their findings correctly, Drs. W. J. Mead, R. D. Evans, and C. Goodman, Massachusetts Institute of Technology scientists told the Geological Society of America.

An "atomic clock," run by the helium gas, given off at a known and constant rate by radioactive minerals, has for some years been used as a clue to the age of rocks. Recently, this method has given a number of answers that appear to be wrong.

Drs. Mead, Evans and Goodman pointed out that wrong answers will always be found if there was any helium in the rocks at the time of formation, or if helium was added to or escaped from the rocks after solidification. A radioactive clock will only tell a time-lapse correctly if it starts at zero, and is not set forward or backward during the time it is running.

Systematic studies of many samples from a single rock mass, close cooperation between physicists, chemists, and geologists, and the adoption of a more scientific attitude toward the results of helium analyses, are necessary, the scientists said, before these "atomic clocks" can be made to give "acceptably reliable" answers.

A NOTE ON ERRORS RESULTING FROM TRANSPOSITION OF DIGITS

CECIL B. READ
University of Wichita, Wichita, Kansas

It is well known that if accounts are off balance by an amount divisible by nine, the error is probably one of transposition of digits. Errors of this type tend to be transposition of adjacent digits. It is by no means as well known that the quotient obtained in the division by nine is of great value in determining what digits were involved in the transposition.

If the tens digit be t and the units digit u, the error resulting from transposition is obviously

$$\pm(10t+u-10u-t)=\pm 9(t-u)$$

that is, the tens and units digits differ by the amount of the quotient obtained in dividing the error by nine. For example, if accounts fail to balance by 63, we find a quotient of 7. If the error be one of transposition, it must have resulted from transposing 0 to 7, 1 and 8, or 2 and 9. Likewise a difference of 72 could result only from transposing 0 and 8 or 1 and 9. An error of 45 could involve only 0 and 5, 1 and 6, 2 and 7, 3 and 8, or 4 and 9.

This simple rule often greatly simplifies the location of errors. Its extension to the cases of such errors as \$63, \$720, or \$2.70 is obvious.

REQUESTS FOR ASSISTANCE

Recently the Secretary of the Association has received requests for materials from two foreign countries. The Embassy of Soviet Russia has asked for materials on the teaching of science in the schools of America and information available is being sent. Parts of the letter from China appear below. You can help if you wish.

"With a view to supplying students with scientific publications to further their research, the National Library takes pleasure in soliciting your assistance. Word has it that assistance rendered at its time of greatest need will forever stand in the memories of individuals who are craving for help. Owing to our hurried departure from Nanking, over 200,000 volumes fell into the hands of the Japanese who set them together with collections of other institutions on fire. With the bombing of Chinese educational institutions by Japanese airplanes, the destruction of Chinese culture is complete. However, the Library has managed to bring a large portion of our holdings to Chungking. It is my crusading responsibility and pleasure to write for complete sets of your publications (SCHOOL SCIENCE AND MATHEMATICS) to be made accessible to our Library. Through a system of mail services, magazines and books thus acquired will be sent to any individual for stated periods of time in any part of free China. In sending us your publications, please send them by book post Via Haiphong, French Indo China. I have the honor to be Yours yours very truly, Chiang Futsung, Librarian."

If you have files of SCHOOL SCIENCE AND MATHEMATICS which you wish to contribute, please send them to me. The Association will box them and ship them Chungking. Please send them by June 1, 1940.

HAROLD H. METCALF,
Oak Park and River Forest High School,
Oak Park, Illinois.

HABITUAL DRUNKARDS LIKELY TO SUFFER FATAL INFLAMMATION OF PANCREAS

The aftermath of a prolonged bout of heavy drinking is likely to be an inflammation of the pancreas which may result in death, Dr. Eugene Clark, assistant medical examiner, New York City, warned at the meeting of the American Association of Pathologists and Bacteriologists.

The disease is more likely to occur in habitual drunkards than in those who occasionally become intoxicated, Dr. Clark found. It bears "as striking a relationship to long continued heavy alcoholic indulgence as does cirrhosis of the liver," he said.

He reported details of the condition of the pancreas at death in 44 cases of acute and chronic alcoholism examined at autopsy in the laboratories of Bellevue Hospital and the office of the Chief Medical Examiner, New York City. In 19 of these the pancreas condition was so severe as to be held responsible for the deaths. A history of long continued heavy indulgence in alcohol was obtained in 11 of these and in at least nine cases the onset of the pancreas disease was preceded by an alcoholic bout. The frequency with which inflammation of the pancreas and alcoholism are associated is indicated from the fact that 27 of the reported cases were found in about 150 consecutively autopsied cases of acute and chronic alcoholism dying in Bellevue Hospital.

The pancreas condition is not significantly associated with cirrhosis of the liver, Dr. Clark said. Cirrhosis of the liver was absent as often as it was present in the cases of the pancreas disease.

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This popular text, teachers find, makes it "*easy to select work for the bright pupil as well as for the dull one.*" The book is effectively organized into three main parts and is sufficiently flexible to permit at least three major adaptations. Giving algebra meaning and vitality, the treatment is motivated by eight descriptive sketches showing the role of mathematics in modern life. A complete reviewing and testing program is included. $1.36

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NAZI ALUMINUM INDUSTRY

Although Germany leads the world in aluminum production (200,000 tons annually) the Reich is suffering from lack of sufficient coal for generating the enormous amounts of electricity needed in the refinement of this light metal, according to official reports received at the U. S. Bureau of Mines here from Germany. It takes 25,000 kilowatt hours of electricity to produce a metric ton of aluminum.

PRESSURE CABINS PERMIT SWIFT DESCENT IN PLANES

In ordinary transport planes a descent which occurs more rapidly than 300 feet per minute has been found to be objectionable to passengers. With the new supercharged cabins on modern transports it is possible for a pilot to descend at 900 feet per minute, if need be, without passenger discomfort, according to Walter Forster of the Curtiss-Wright Corporation in a report to the Society of Automotive Engineers.

Just Published

BIOLOGY IN THE MAKING

By Emily Eveleth Snyder

Readable, interesting background material for biology classes, tracing the development of the most important fields of biology from Aristotle to the present, with emphasis on those scientists who have made outstanding contributions to biological progress. The non-technical vocabulary is within the comprehension of 10th grade students. $2.80

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